Midterm #1

MATH 332 — Differential Equations II
Wednesday, Oct 2, 2024

Please print your name:

No notes, fancy calculators or tools of any kind are permitted. There are 30 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (3 points) Consider the following system of initial value problems:

$$\begin{array}{l} y_1'' - 2y_1 = 3y_2' \\ y_2'' + 5y_2 = 4y_1' \end{array} \qquad y_1(0) = 0,\ y_1'(0) = 7,\ y_2(0) = -2,\ y_2'(0) = 6$$

Write it as a first-order initial value problem in the form $\boldsymbol{y}' = M\boldsymbol{y}$, $\boldsymbol{y}(0) = \boldsymbol{c}$.

Problem 2. (1+4+1 points) Consider the sequence a_n defined by $a_{n+2} = a_{n+1} + 6a_n$ and $a_0 = 3$, $a_1 = 4$.

(a) The next two terms are $a_2 =$ ____ and $a_3 =$ ____.

(b) A Binet-like formula for a_n is $a_n =$ ____, and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} =$ ____.

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Problem 3. (7+1+2 points) Let $M = \begin{bmatrix} -1 & 5 \\ 1 & 3 \end{bmatrix}$.

(a) Compute e^{Mt}.

(b) Solve the initial value problem $\boldsymbol{y}' = M\boldsymbol{y}$ with $\boldsymbol{y}(0) = \begin{bmatrix} 0 \\ 6 \end{bmatrix}$.

(c) Determine all equilibrium points of $\begin{bmatrix} x \\ y \end{bmatrix}' = M \begin{bmatrix} x \\ y \end{bmatrix}$ and their stability.

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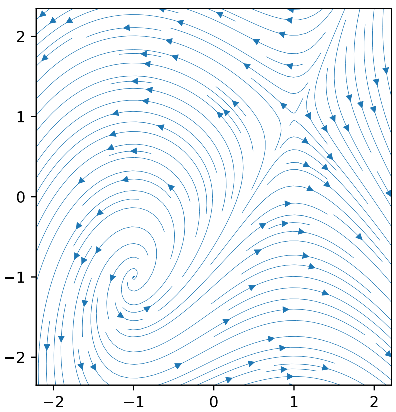
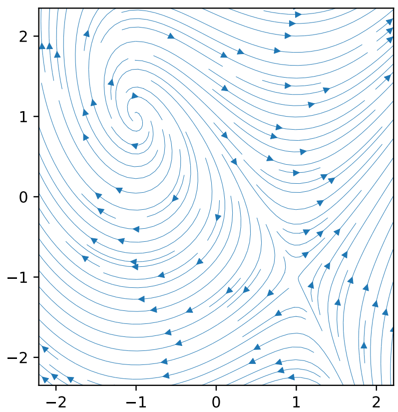
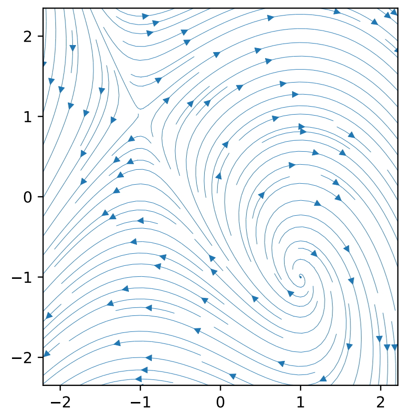
Problem 4. (1+2 points)

(a) Circle the phase portrait below which belongs to $\frac{\mathrm{d}x}{\mathrm{d}t} = x + y$, $\frac{\mathrm{d}y}{\mathrm{d}t} = x^2 - 1$.

(b) Determine all equilibrium points and classify the stability of each.







Problem 5. (2+2+2+2 points) Fill in the blanks. None of the problems should require any computation!

(a) If $M^n = \begin{bmatrix} 2 \cdot 2^n - 1 & -2 \cdot 2^n + 2 \\ 2^n - 1 & -2^n + 2 \end{bmatrix}$, then $e^{Mx} =$ ________.

(b) Determine a (homogeneous linear) recurrence equation satisfied by $a_n = (4n-1)5^n + 7n^2$.

You can use the operator N to write the recurrence. No need to simplify, any form is acceptable.

(c) Let y_p be any solution to the inhomogeneous linear differential equation $y'' + 2y = 3x^2 e^{4x} + 5$. Find a homogeneous linear differential equation which y_p solves.

You can use the operator D to write the DE. No need to simplify, any form is acceptable.

(d) Consider a homogeneous linear differential equation with constant real coefficients which has order 5. Suppose $y(x) = 4e^x \sin(3x) + 5x^2$ is a solution. Write down the general solution.

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(extra scratch paper)