Midterm #2

MATH 332 — Differential Equations II
Wednesday, Nov 15, 2023

Please print your name:

No notes, calculators or tools of any kind are permitted. There are 31 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (7 points) Determine the equilibrium points of the system $\frac{dx}{dt} = (x-2)y$, $\frac{dy}{dt} = xy - 1$ and classify their stability.

Problem 2. (3 points) A mass-spring system is described by the equation $my'' + 2y = \sum_{n=1}^{\infty} \frac{1}{2n^2} \cos\left(\frac{nt}{3}\right)$.

For which values of m does resonance occur?

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Problem 3. (3 points) Let $y(x)$ be the unique solution to the IVP $y'' = 1 + 2(x-1)y^2$, $y(0) = 1$, $y'(0) = 2$. Determine the first several terms (up to x^3) in the power series of $y(x)$.

Problem 4. (3 points) Find a minimum value for the radius of convergence of a power series solution to

$$(x-3)y'' = \frac{2y+1}{x^2+1} \quad \text{at } x = 1.$$

Problem 5. (6 points) Derive a recursive description of a power series solution $y(x)$ (around $x=0$) to the differential equation $y'' = x^2 y' + 3y$.

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Problem 6. (5 points)

(a) Suppose $y(x) = \sum_{n=0}^{\infty} a_n (x+2)^n$. How can we compute the a_n from $y(x)$? $a_n =$ ______ .

(b) Suppose $f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(3n\pi t) + b_n \sin(3n\pi t))$. How can we compute the a_n and b_n from $f(t)$?

$a_n =$ ______ and $b_n =$ ______ .

(c) Determine the power series around $x = 0$: $\frac{3}{1+7x} =$ ______

(d) Determine the power series around $x = 0$: $e^{-3x} =$ ______

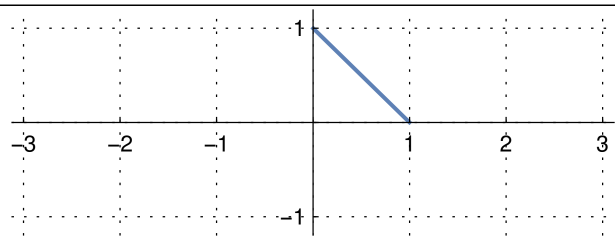
Problem 7. (4 points) Consider the function $f(t) = 1 - t$, defined for $t \in [0, 1]$.

(a) Sketch the Fourier series of $f(t)$ for $t \in [-3, 3]$.

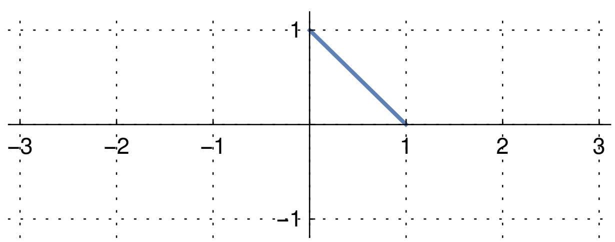
(b) Sketch the Fourier cosine series of $f(t)$ for $t \in [-3, 3]$.

(c) Sketch the Fourier sine series of $f(t)$ for $t \in [-3, 3]$.

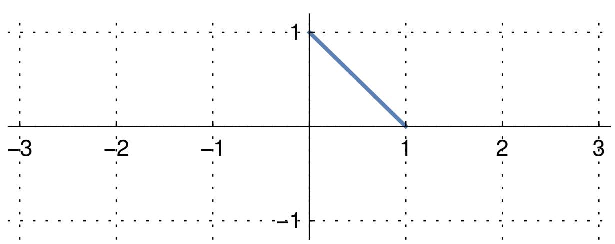
In each sketch, carefully mark the values of the Fourier series at discontinuities.



(a)



(b)



(c)

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(extra scratch paper)