Midterm #1

MATH 332 — Differential Equations II
Wednesday, Oct 4, 2023

Please print your name:

No notes, calculators or tools of any kind are permitted. There are 30 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (10 points) Let $M = \begin{bmatrix} 1 & 3 \\ -1 & 5 \end{bmatrix}$.

(a) Compute e^{Mt}.

(b) Solve the initial value problem $\boldsymbol{y}' = M\boldsymbol{y}$ with $\boldsymbol{y}(0) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$.

(c) Determine all equilibrium points of $\begin{bmatrix} x \\ y \end{bmatrix}' = M \begin{bmatrix} x \\ y \end{bmatrix}$ and their stability.

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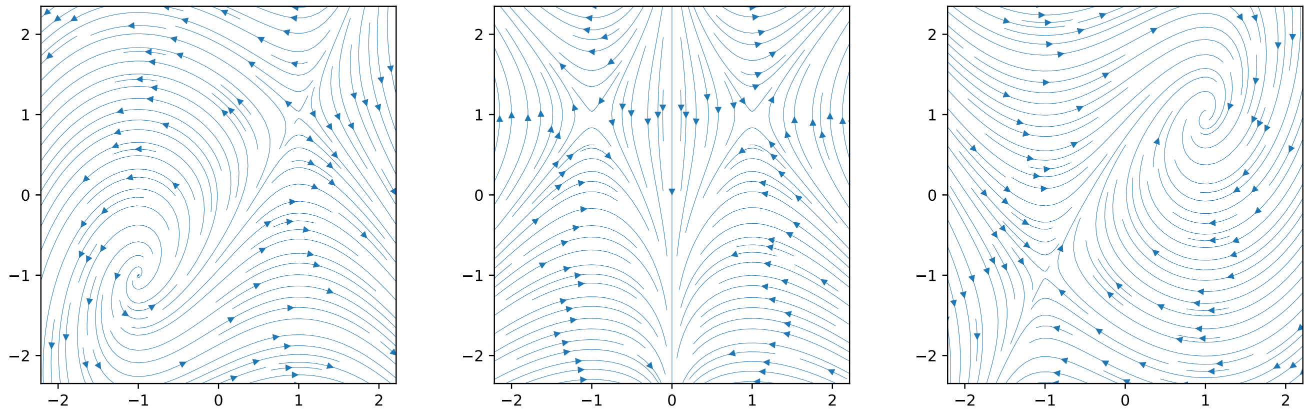
Problem 2. (3 points)

(a) Circle the phase portrait below which belongs to $\frac{\mathrm{d}x}{\mathrm{d}t} = y - x$, $\frac{\mathrm{d}y}{\mathrm{d}t} = 1 - x^2$.

(b) Determine all equilibrium points and classify the stability of each.



Problem 3. (8 points) Fill in the blanks. None of the problems should require any computation!

(a) Consider a homogeneous linear differential equation with constant real coefficients which has order 4. Suppose $y(x) = 7x + 2e^{-x}\cos(2x)$ is a solution. Write down the general solution.

(b) Let y_p be any solution to the inhomogeneous linear differential equation $y'' - 4y = 3e^x + 5x^2$. Find a homogeneous linear differential equation which y_p solves.

You can use the operator D to write the DE. No need to simplify, any form is acceptable.

(c) Determine a (homogeneous linear) recurrence equation satisfied by $a_n = (n+3)4^n + 5$.

You can use the operator N to write the recurrence. No need to simplify, any form is acceptable.

(d) If $e^{Mx} = \begin{bmatrix} 2e^{2x} - e^x & -2e^{2x} + 2e^x \\ e^{2x} - e^x & -e^{2x} + 2e^x \end{bmatrix}$, then $M^n =$ ________.

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Problem 4. (3 points) Consider the following system of initial value problems:

$$\begin{aligned} y_1'' &= 4y_1 + 3y_2 \\ y_2'' &= 5y_1' - 2y_2' \end{aligned} \qquad y_1(0)=1,\ y_1'(0)=0,\ y_2(0)=6,\ y_2'(0)=7$$

Write it as a first-order initial value problem in the form $\boldsymbol{y}' = M\boldsymbol{y},\ \boldsymbol{y}(0) = \boldsymbol{c}$.

Problem 5. (1+4+1 points) Consider the sequence a_n defined by $a_{n+2} = a_{n+1} + 2a_n$ and $a_0 = 0,\ a_1 = 6$.

(a) The next two terms are $a_2 =$ ☐ and $a_3 =$ ☐.

(b) A Binet-like formula for a_n is $a_n =$ ☐, and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} =$ ☐.

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(extra scratch paper)

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