Example of solving system of DEs

review $\quad y' = My$

- if v is λ-eigenvector of M, then a solution is $y = e^{\lambda x} v$
- e^{Mx} is unique solution to $\Phi' = M\Phi$, $\Phi(0) = I$
  matrix exponential
- $e^{Mx} = \Phi(x)\,\Phi(0)^{-1}$ for any fundamental matrix solution Φ

EG $\quad M = \begin{bmatrix} 8 & -10 \\ 5 & -7 \end{bmatrix}$

(a) general solution to $y' = My$

$\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ 3-eigenvector $\qquad \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ -2-eigenvector

$$c_1 e^{3x}\begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 e^{-2x}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \Phi \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

(b) fundamental matrix solution to $y' = My$

$$\Phi(x) = \begin{bmatrix} 2e^{3x} & e^{-2x} \\ e^{3x} & e^{-2x} \end{bmatrix}$$

(c) compute e^{Mx}:

$$e^{Mx} = \Phi(x)\,\Phi(0)^{-1}$$

$$= \begin{bmatrix} 2e^{3x} & e^{-2x} \\ e^{3x} & e^{-2x} \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^{-1}$$

$$\frac{1}{1}\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2e^{3x} - e^{-2x} & -2e^{3x} + 2e^{-2x} \\ e^{3x} - e^{-2x} & -e^{3x} + 2e^{-2x} \end{bmatrix}$$

check:
$x = 0$: $\begin{bmatrix} 2-1 & -2+2 \\ 1-1 & -1+2 \end{bmatrix} = I$

check: $\frac{d}{dx} e^{Mx} = M e^{Mx}$

HW!