Homogeneous LDEs with constant coefficients — part 2

can be written as $p(D)\,y = 0$ $\qquad D := \frac{d}{dx}$

characteristic polynomial

EG $y''' + 7y'' + 14y' + 8y = 0$

$p(D)y = 0 \qquad p(D) = D^3 + 7D^2 + 14D + 8$

$= (D+1)(D+2)(D+4)$

general solution:

$C_1 e^{-x} + C_2 e^{-2x} + C_3 e^{-4x}$

EG $y'' - 4y' + 13y = 0$

$p(D) = D^2 - 4D + 13 \qquad$ roots: $2+3i\,,\ 2-3i$

general solution:

$C_1 e^{(2+3i)x} + C_2 e^{(2-3i)x}$

$e^{2x}[\cos(3x) - i\sin(3x)]$

Euler: $e^{2x} e^{3ix} = e^{2x}[\cos(3x) + i\sin(3x)]$

real-term general solution:

$D_1 e^{2x}\cos(3x) + D_2 e^{2x}\sin(3x)$