Net ID: *Name:*

Midterm #2

MATH 286 — Differential Equations Plus

Thursday, March 13

- No notes, personal aids or calculators are permitted.
- Answer all questions in the space provided. If you require more space to write your answer, you may continue on the back of the page. There is a blank page at the end of the exam for rough work.
- **Explain your work!** Little or no points will be given for a correct answer with no explanation of how you got it.

Good luck!

Problem 1. (5 points) Consider a homogeneous linear differential equation with constant real coefficients which has order 6. Suppose $y(x) = x^2 e^{2x} \cos(x)$ is a solution. Write down the general solution.

$y(x) =$

Problem 2. (20 points) Find the general solution of $\boldsymbol{x}' = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \boldsymbol{x}$.

$\boldsymbol{x}(t) =$

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Problem 3. (10 points) The position $x(t)$ of a certain mass on a spring is described by $x'' + cx' + 5x = F\sin(\omega t)$.

(a) Assume first that there is no external force, i.e. $F = 0$. For which values of c is the system overdamped?

(b) Now, $F \neq 0$ and the system is undamped, i.e. $c = 0$. For which values of ω, if any, does resonance occur?

Overdamped for:	Resonance for:

Problem 4. (20 points) Find the general solution of the differential equation $y^{(3)} - y = e^x + 7$.

$y(x) =$

Problem 5. (20 points) Consider, for $x > 0$, the second-order differential equation

$$y'' - \left(1 + \frac{2}{x}\right) y' + \left(\frac{1}{x} + \frac{2}{x^2}\right) y = 0.$$

(a) Show that the the functions $y_1(x) = x$ and $y_2(x) = x\, e^x$ are solutions to this differential equation.

(b) Using the Wronskian, show that y_1 and y_2 are linearly independent solutions to the above differential equation.

(c) Find, for $x > 0$, the general solution to the second-order differential equation

$$y'' - \left(1 + \frac{2}{x}\right) y' + \left(\frac{1}{x} + \frac{2}{x^2}\right) y = 2x.$$

$y(x) =$

Problem 6. (20 points) The motion of a certain mass on a spring is described by $x'' + 2x' + 2x = 5\sin(t)$.

(a) What is the amplitude of the resulting steady periodic oscillations?

(b) Assume that the mass is initially at rest (i.e. $x(0) = 0$, $x'(0) = 0$) and find the position function $x(t)$.

Amplitude:	$x(t) =$

Problem 7. (5 points) Let y_p be any solution to the inhomogeneous linear differential equation $y'' + xy = e^x$. Find a homogeneous linear differential equation which y_p solves. *Hint:* Do not attempt to solve the DE.

Homogeneous linear DE:

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