Preparing Midterm #2

MATH 286 — Differential Equations Plus
March 13, 2014

Problem 1. Solve $y'' + 2y' + y = 2e^{2x} + e^{-x}$, $y(0) = -1$, $y'(0) = 2$.

Problem 2.

(a) Assume that the angle $\theta(t)$ of a swinging pendulum is described by $\theta'' + 4\theta = 0$. Suppose $\theta(0) = \frac{3}{10}$ and $\theta'(0) = -\frac{4}{5}$. What is the amplitude of the resulting periodic oscillations?

(b) For which values of the damping constant $c > 0$ is the system $y'' + cy' + 5y = 0$ underdamped?

(c) For which value of the external frequency ω does the system $y'' + 4y = 3\cos(\omega x)$ exhibit resonance?

(d) A forced mechanical oscillator is described by $x'' + 2x' + x = 25\cos(2t)$. What is the amplitude of the resulting steady periodic oscillations?

Problem 3. The motion of a certain mass on a spring is described by $x'' + x' + \frac{x}{2} = 5\sin(\omega t)$.

(a) Assume first that $\omega = 1$. Find the position function $x(t)$ if $x(0) = 2$ and $x'(0) = 0$.

(b) Suppose the frequency of the external force is changed so that $x'' + x' + \frac{x}{2} = 5\sin(\omega t)$. Does practical resonance occur for some value of ω? If so, for what ω?

Problem 4. Find the general solution of $y'' - 4y' + 4y = 3e^{2x}$.

Problem 5.

(a) Consider the differential equation $x^2y'' - 4xy' + 6y = 0$. Find all solutions of the form $y(x) = x^r$.

(b) Show that the solutions you found are independent.

(c) Note that the Wronskian of your solutions is zero for $x = 0$. Why does this not contradict the independence?

(d) Find the general solution of $x^2y'' - 4xy' + 6y = x^3$.

Problem 6. Solve $\boldsymbol{x}' = \begin{pmatrix} 3 & -2 & 0 \\ -1 & 3 & -2 \\ 0 & -1 & 3 \end{pmatrix}\boldsymbol{x}$, $\boldsymbol{x}(0) = \begin{pmatrix} 0 \\ 2 \\ 6 \end{pmatrix}$.

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