Review

- Basic understanding
  - DEs and IVPs
  - existence and uniqueness
  - visualization of first-order DEs via slope fields
- Basic modeling
  - population models
  - modeling simple motions
  - mixing problems
- Solving techniques
  - linear DEs with constant coefficients
  - separation of variables ($y' = f(x)g(y)$)
  - linear first-order equations (integrating factor)
  - common substitutions (e.g. $y' = f(y/x)$)

Example 62. What can we say about existence and uniqueness of the initial value problem $\frac{\mathrm{d}y}{\mathrm{d}x} = \ln(x^2 + y^2) - \frac{\cos(y^3)}{x}$, $y(1) = 0$?

Solution. Write as $y' = f(x, y)$ with $f(x, y) = \ln(x^2 + y^2) - \frac{\cos(y^3)}{x}$. Then $\frac{\partial}{\partial y} f(x, y) = \frac{2y}{x^2 + y^2} + \frac{3y^2 \sin(y^3)}{x}$. We observe that both $f(x, y)$ and $\frac{\partial}{\partial y} f(x, y)$ are continuous for all (x, y) with $x \neq 0$. In particular, $f(x, y)$ and $\frac{\partial}{\partial y} f(x, y)$ are continuous around $(1, 0)$. Consequently, the IVP has a solution and it is unique. ◇

Example 63. $x^2(2y^3 - y)\frac{\mathrm{d}y}{\mathrm{d}x} = xy^5 - y^5$

Solution. This equation is separable! (Note that $xy^5 - y^5 = y^5(x - 1)$.) ◇

Example 64. $xy' = 4x^4 - (2x - 3)y$

Solution. This equation is linear! Write as $y' + \frac{2x - 3}{x} y = 4x^3$, determine integrating factor, ... ◇

Example 65. $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{xy + \sec^2(x)\cos(y) + x\cos(y) + y\sec^2(x)}{1 - \sin(y)}$

Solution. This equation is separable! $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(x + \sec^2(x))(y + \cos(y))}{1 - \sin(y)}$ ◇

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