Midterm #1

MATH 238 — Differential Equations I
Monday, Sept 29, 2025

Please print your name:

No notes, calculators or tools of any kind are permitted. There are 29 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (4 points) A rising population is modeled by the equation $\frac{\mathrm{d}P}{\mathrm{d}t} = 600P - 2P^2$. Answer the following questions without solving the differential equation.

(a) When the population size stabilizes in the long term, how big will the population be?

(b) What is the population size when it is growing the fastest?

Problem 2. (4 points) Consider the IVP $\frac{\mathrm{d}y}{\mathrm{d}x} = x + 2y$ with $y(1) = 0$. Approximate the solution $y(x)$ for $x \in [1, 2]$ using Euler's method with 2 steps. In particular, what is the approximation of $y(2)$?

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Problem 3. (8 points) A tank contains 10gal of pure water. It is filled with brine (containing 4lb/gal salt) at a rate of 2gal/min. At the same time, well-mixed solution flows out at a rate of 1gal/min. How much salt is in the tank after t minutes?

Problem 4. (2 points) In the differential equation $(3x+1)\frac{\mathrm{d}y}{\mathrm{d}x} = 4y - \sin\left(1 + \frac{y}{x^2}\right)$ substitute $u = 1 + \frac{y}{x^2}$.
What is the resulting differential equation for u?

No need to simplify! Do not attempt to solve!

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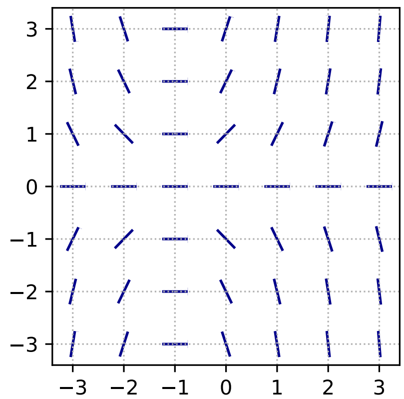
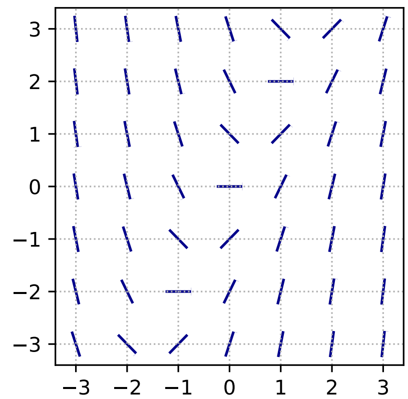
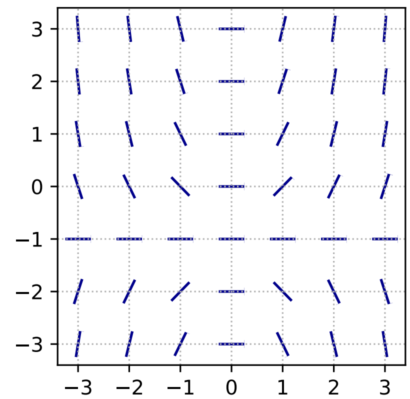
Problem 5. (2 points) Circle the slope field below which belongs to the differential equation $y' = (y+1)x$.







Problem 6. (6 points) Find a general solution to the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} + 1 = \sqrt{x+y}$.

Problem 7. (3 points) Consider the initial value problem $(4 - x^2)y' = 3\cos(5y^2 + 1)$, $y(a) = b$. For which values of a and b can we guarantee existence and uniqueness of a (local) solution?

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(extra scratch paper)

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