Midterm #2

Please print your name:

No notes, calculators or tools of any kind are permitted.   There are 33 points in total.   You need to show work to receive full credit.

Good luck!

Problem 1. (10 points) Determine the general solution of the system $\begin{aligned} y_1' &= y_1 + y_2 \\ y_2' &= 3y_1 - y_2 + 6e^x \end{aligned}$.

Problem 2. (5 points) The position $y(t)$ of a certain mass on a spring is described by $2y'' + dy' + 4y = F\sin(\omega t)$.

(a) Assume first that there is no external force, i.e. $F = 0$. For which values of d is the system underdamped?

(b) Now, $F \neq 0$ and the system is undamped, i.e. $d = 0$. For which values of ω, if any, does resonance occur?
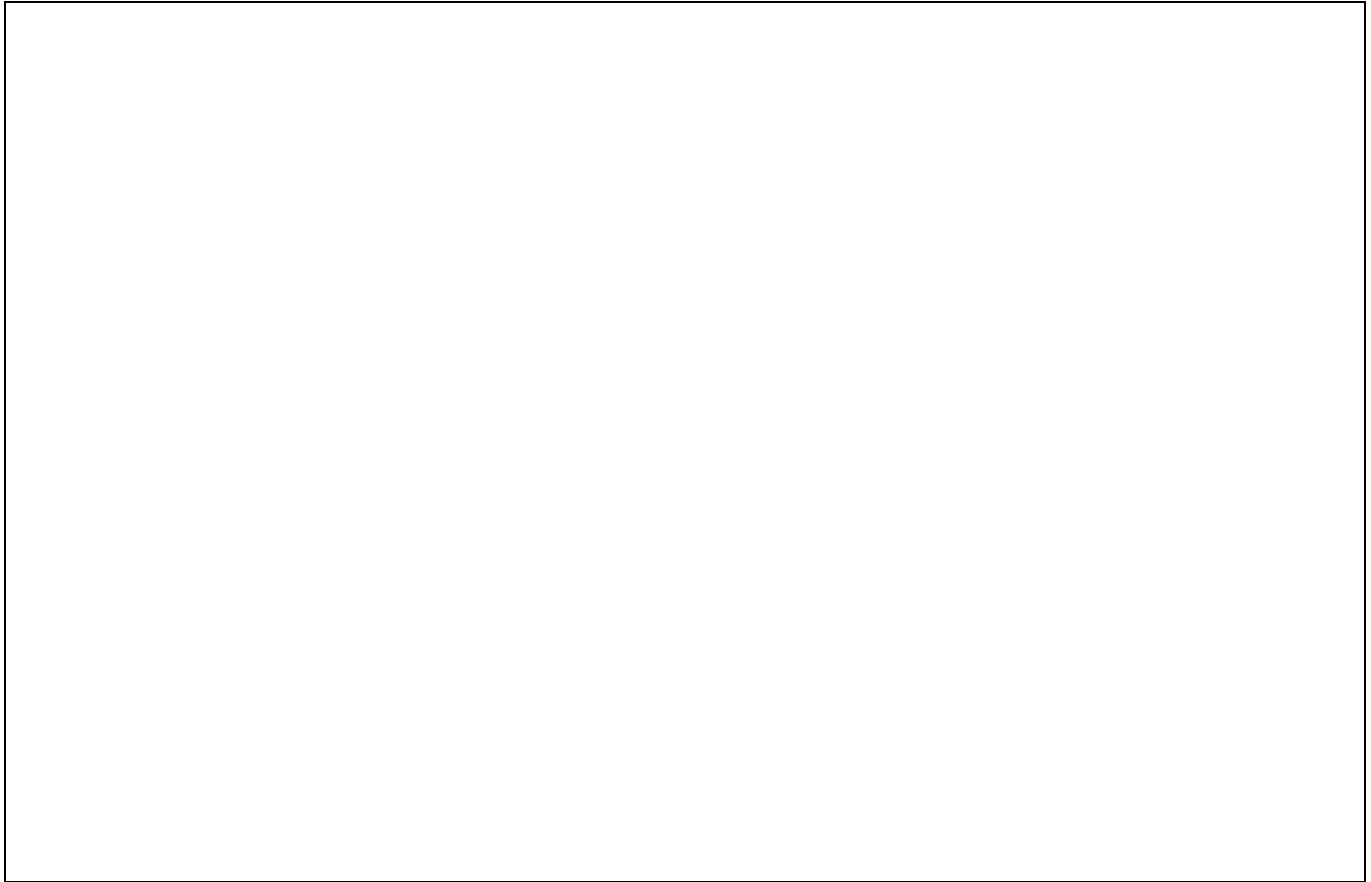
Problem 3. (5 points) Let L be a linear differential operator of order 4 with constant real coefficients. Suppose that $1 - 2i$ is a repeated characteristic root of L.

(a) What is the general solution to $Ly = 0$?

(b) Write down the simplest form of a particular solution y_p of the DE $Ly = 2e^x\cos(2x) - 5xe^x$ with undetermined coefficients.
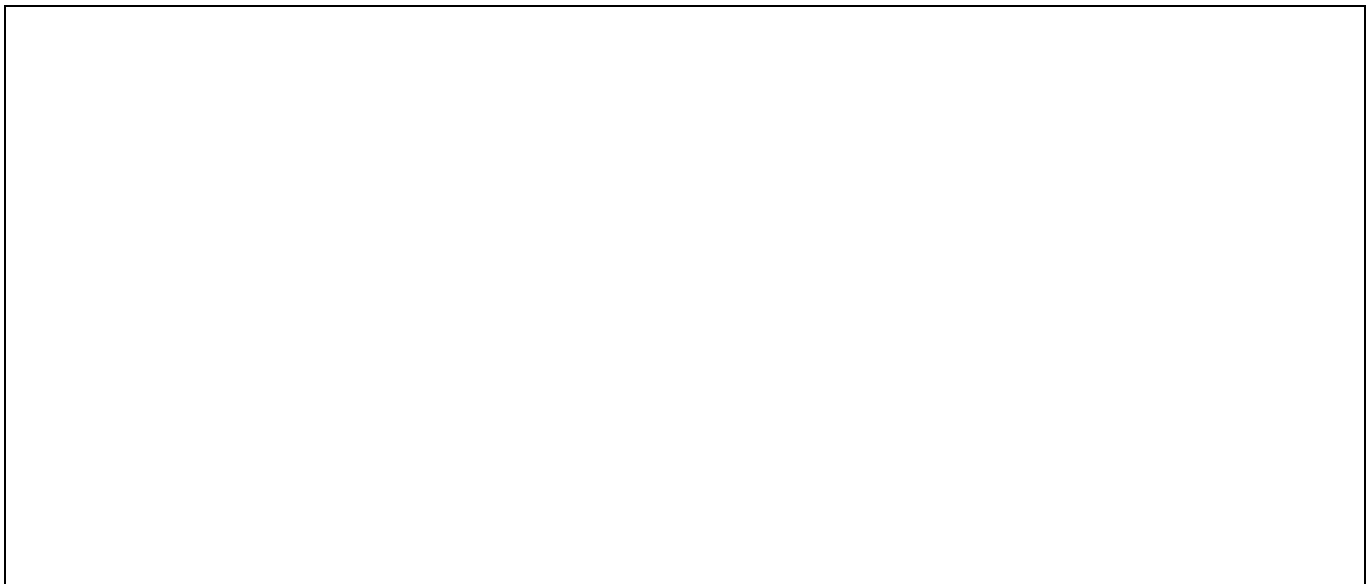
Problem 4. (3 points) Write the (third-order) differential equation $y''' + 7y'' - 5y' - 2y = \cos(2x)$ as a system of (first-order) differential equations.

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Problem 5. (6 points) The mixtures in two tanks T_1, T_2 are kept uniform by stirring. Brine containing 3 lb of salt per gallon enters T_1 at a rate of 3 gal/min, while brine containing 2 lb of salt per gallon enters T_2 at a rate of 4gal/min. Mixed solution from T_1 is pumped into T_2 at a rate of 1 gal/min, and also from T_2 into T_1 at a rate of 2 gal/min. Each tank initially contains 10 gal of pure water.

Denote by $y_i(t)$ the amount (in pounds) of salt in tank T_i at time t (in minutes). Derive a system of linear differential equations for the y_i, including initial conditions. (Do *not* attempt to solve the system.)

Problem 6. (4 points) Assume that the angle $\theta(t)$ of a swinging pendulum is described by $\theta'' + 9\theta = 0$. Suppose $\theta(0) = 2$, $\theta'(0) = -3$. What is the period and the amplitude of the resulting oscillations?

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(extra scratch paper)