Midterm #1

MATH 238 — Differential Equations I
Wednesday, Oct 4, 2023

Please print your name:

No notes, calculators or tools of any kind are permitted. There are 30 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (4 points) A rising population is modeled by the equation $\frac{\mathrm{d}P}{\mathrm{d}t} = 100P - 2P^2$. Answer the following questions without solving the differential equation.

(a) When the population size stabilizes in the long term, how big will the population be?

(b) What is the population size when it is growing the fastest?

Problem 2. (4 points) Consider the IVP $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x - y$ with $y(1) = 2$. Approximate the solution $y(x)$ for $x \in [1, 2]$ using Euler's method with 2 steps. In particular, what is the approximation of $y(2)$?

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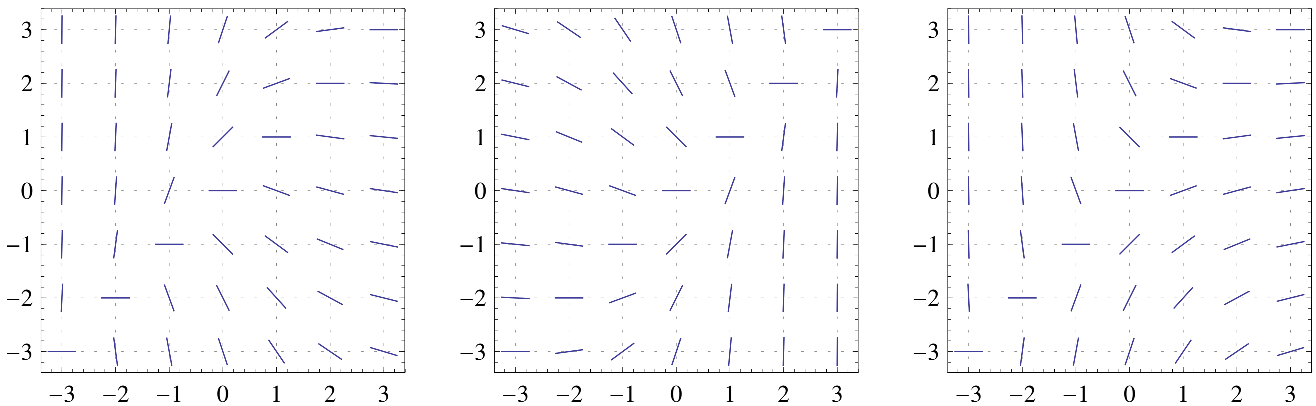
Problem 3. (3 points) Find the general solution to the differential equation $y'' = 2y' + 3y$.

Problem 4. (2 points) Circle the slope field below which belongs to the differential equation $e^x y' = y - x$.



Problem 5. (4 points) Solve the initial value problem $\frac{\mathrm{d}y}{\mathrm{d}x} = 3y^2$ with $y(2) = 1$.

Problem 6. (2 points) In the differential equation $xy\frac{\mathrm{d}y}{\mathrm{d}x} = \sin\left(\frac{y}{x}\right)$ substitute $u = \frac{y}{x}$.
What is the resulting differential equation for u?

No need to simplify! Do not attempt to solve!

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Problem 7. (3 points) Consider the initial value problem $x(y+1)y' + x^2 = 3$, $y(a) = b$. For which values of a and b can we guarantee existence and uniqueness of a (local) solution?

Problem 8. (8 points) A tank contains 5gal of pure water. It is filled with brine (containing 6lb/gal salt) at a rate of 2gal/min. At the same time, well-mixed solution flows out at a rate of 1gal/min. How much salt is in the tank after t minutes?

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(extra scratch paper)