Midterm #1 – Practice

MATH 238 — Differential Equations I
Midterm: Wednesday, Oct 4, 2023

Please print your name:

Bonus challenge. Let me know about any typos you spot in the posted solutions (or lecture sketches). Any mathematical typo, that is not yet fixed by the time you send it to me, is worth a bonus point.

Reminder. No notes, calculators or tools of any kind will be permitted on the midterm exam.

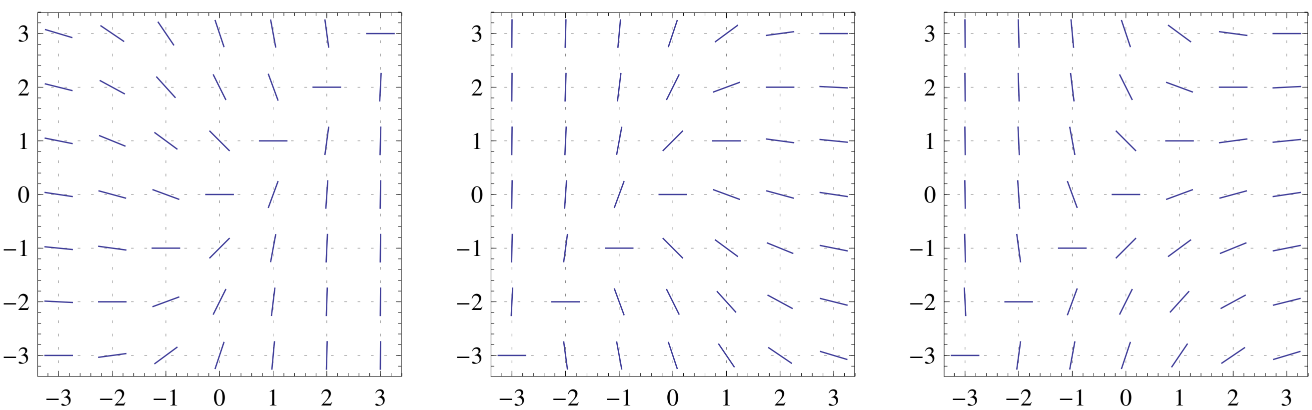
Problem 1. Find the general solution to $y'' + y' = 12y$.

Problem 2. Consider the initial value problem

$$(xy + 2x)y' = \cos(x), \qquad y(a) = b.$$

For which values of a and b can we guarantee existence and uniqueness of a (local) solution?

Problem 3. Circle the slope field below which belongs to the differential equation $e^x y' = x - y$.



Problem 4. Consider the IVP $(2x+1)\frac{\mathrm{d}y}{\mathrm{d}x} = y + 2x + 1$, $y(2) = 0$.

(a) Approximate the solution $y(x)$ for $x \in [2,3]$ using Euler's method with 2 steps. In particular, what is the approximation of $y(3)$?

(b) **(use calculator)** Likewise approximate the solution $y(x)$ for $x \in [2,3]$ using Euler's method with 3 steps.

(c) Solve this IVP exactly. In particular, what is the exact value of $y(3)$? Compare with the approximate values at $x = 3$.

Armin Straub
straub@southalabama.edu

Problem 5. In the differential equation $x(y+1)\frac{\mathrm{d}y}{\mathrm{d}x}=(x^2+y)^3$ substitute $u=(x^2+y)^3$.

What is the resulting differential equation for u? No need to simplify!

Problem 6. Solve the initial value problem $y'=2xy+3x^2e^{x^2}$, $y(0)=5$.

Problem 7. Find a general solution to the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x}+y^2\sin(x)=0$.

Problem 8. Find a general solution to the differential equation $xy'=y+x^2\cos(x)$.

Problem 9. A tank contains 20gal of pure water. It is filled with brine (containing 5lb/gal salt) at a rate of 8gal/min. At the same time, well-mixed solution flows out at a rate of 6gal/min. How much salt is in the tank after t minutes?

Problem 10. The time rate of change of a rabbit population P is proportional to the square root of P. At time $t=0$, the population numbers 100 rabbits and is increasing at the rate of 20 rabbits per month. How many rabbits will there be after two months?

Problem 11. A rising population is modeled by the differential equation $\frac{\mathrm{d}P}{\mathrm{d}t}=1000P-20P^2$.

(a) When the population size stabilizes in the long term, how big will the population be?

(b) Under which condition will the population size shrink?

(c) What is the population size when it is growing the fastest?

Problem 12. Solve the initial value problem $x\frac{\mathrm{d}y}{\mathrm{d}x}=y-xe^{y/x}$, $y(1)=0$.

Problem 13. Solve the initial value problem $(x^2+1)\frac{\mathrm{d}y}{\mathrm{d}x}+xy=\frac{1}{\sqrt{x^2+1}}$, $y(0)=1$.

Problem 14. Find a general solution to the differential equation $2+\frac{\mathrm{d}y}{\mathrm{d}x}=\sqrt{2x+y}$.