

Midterm #2

Please print your name:

No notes, calculators or tools of any kind are permitted. There are 42 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (7 points) Determine the general solution $y(x)$ to the differential equation $y^{(4)} + 4y''' + 5y'' = 2$. Express the solution using real numbers only.

$y(x) =$

Problem 2. (3 points) Consider a homogeneous linear differential equation with constant real coefficients which has order 6. Suppose $y(x) = 3x^2e^{2x}\cos(x)$ is a solution. Write down the general solution.

$y(x) =$

Problem 3. (10 points) Determine the general solution of the system
$$\begin{aligned} y_1' &= 3y_1 + y_2 \\ y_2' &= y_1 + 3y_2 + 6e^x. \end{aligned}$$

$$y_1(x) =$$

$$y_2(x) =$$

Problem 4. (3 points) The position $y(t)$ of a certain mass on a spring is described by $2y'' + dy' + 3y = 0$. For which value of d is the system critically damped?

Critically damped for $d =$

Problem 5. (3 points) Write the (third-order) differential equation $y''' + 4y'' - 2y' - 3y = e^x$ as a system of (first-order) differential equations.

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Problem 6. (6 points) Let L be a linear differential operator of order 4 with constant real coefficients. Suppose that $2 - 3i$ is a repeated characteristic root of L .

- (a) What is the general solution to $Ly = 0$?
- (b) Write down the simplest form of a particular solution y_p of the DE $Ly = -4x^2e^{3x}$ with undetermined coefficients.
- (c) Write down the simplest form of a particular solution y_p of the DE $Ly = -4e^{2x}\sin(3x)$ with undetermined coefficients.

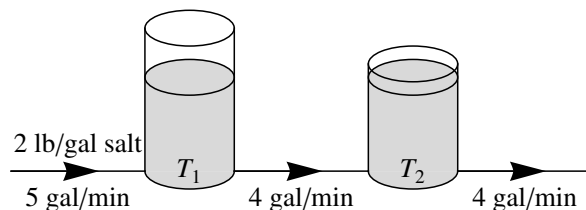
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Problem 7. (6 points) The mixtures in two tanks T_1, T_2 are kept uniform by stirring. The tanks are connected as indicated in the sketch below. That is, brine containing 2 lb of salt per gallon enters T_1 at 5 gal/min, and the solution is pumped at a rate of 4 gal/min into T_2 . Finally, solution is leaving T_2 at 4 gal/min.

Initially, T_1 and T_2 contain 40gal of pure water each.

Denote by $y_i(t)$ the amount (in pounds) of salt in tank T_i at time t (in minutes). Derive a system of linear differential equations for the y_i , including initial conditions.

(Do *not* attempt to solve the system.)



Problem 8. (4 points) Assume that the angle $\theta(t)$ of a swinging pendulum is described by $9\theta'' + \theta = 0$. Suppose $\theta(0) = \frac{1}{10}$, $\theta'(0) = -\frac{1}{3}$. What is the period and the amplitude of the resulting oscillations?

period:	amplitude:
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(extra scratch paper)