

# Midterm #2 – Practice

Please print your name:

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**Reminder.** No notes, calculators or tools of any kind will be permitted on the midterm exam.

**Problem 1.** Let  $L$  be a linear differential operator of order 4 with constant real coefficients. Suppose that  $3 + 7i$  is a repeated characteristic root of  $L$ .

- What is the general solution to  $Ly = 0$ ?
- Write down the simplest form of a particular solution  $y_p$  of the DE  $Ly = 7x^2e^{3x}$  with undetermined coefficients.
- Write down the simplest form of a particular solution  $y_p$  of the DE  $Ly = e^{3x}\sin(7x)$  with undetermined coefficients.

**Problem 2.** Consider a homogeneous linear differential equation with constant real coefficients which has order 8.

- Suppose  $y(x) = 7x - 2x^2e^{3x}\sin(5x)$  is a solution. Write down the general solution.
- Suppose  $y(x) = 2xe^{3x} + x\cos(5x) - 5\sin(x)$  is a solution. Write down the general solution.

**Problem 3.**

- Determine the general solution of the system 
$$\begin{aligned} y_1' &= y_1 - 6y_2 \\ y_2' &= y_1 - 4y_2 \end{aligned}$$
- Solve the IVP 
$$\begin{aligned} y_1' &= y_1 - 6y_2 \\ y_2' &= y_1 - 4y_2 \end{aligned} \quad \text{with} \quad \begin{aligned} y_1(0) &= 4 \\ y_2(0) &= 1 \end{aligned}$$
- Determine a particular solution to 
$$\begin{aligned} y_1' &= y_1 - 6y_2 \\ y_2' &= y_1 - 4y_2 - 2e^{3x} \end{aligned}$$
- Determine the general solution to 
$$\begin{aligned} y_1' &= y_1 - 6y_2 \\ y_2' &= y_1 - 4y_2 - 2e^{3x} \end{aligned}$$

**Problem 4.**

- Write the (third-order) differential equation  $y''' + 2y'' - 4y' + 5y = 2\sin(x)$  as a system of (first-order) differential equations.
- Consider the following system of (second-order) initial value problems:

$$\begin{aligned} y_1'' &= 5y_1' + 2y_2' + e^{2x} \\ y_2'' &= 7y_1 - 3y_2 - 3e^x \end{aligned} \quad y_1(0) = 1, \quad y_1'(0) = 4, \quad y_2(0) = 0, \quad y_2'(0) = -1$$

Write it as a first-order initial value problem in the form  $\mathbf{y}' = M\mathbf{y}$ ,  $\mathbf{y}(0) = \mathbf{c}$ .

**Problem 5.**

- (a) Determine the general solution to  $y'' - 4y' + 4y = 3e^{2x}$ .
- (b) Determine the general solution to the differential equation  $y''' - y = e^x + 7$ .
- (c) Determine the general solution  $y(x)$  to the differential equation  $y^{(4)} + 6y''' + 13y'' = 2$ . Express the solution using real numbers only.
- (d) Solve the initial value problem  $y'' + 2y' + y = 2e^{2x} + e^{-x}$ ,  $y(0) = -1$ ,  $y'(0) = 2$ .

**Problem 6.** The mixtures in three tanks  $T_1, T_2, T_3$  are kept uniform by stirring. Brine containing 2 lb of salt per gallon enters the first tank at 15 gal/min. Mixed solution from  $T_1$  is pumped into  $T_2$  at 10 gal/min and from  $T_2$  into  $T_3$  at 10 gal/min. Each tank initially contains 10 gal of pure water. Denote by  $y_i(t)$  the amount (in pounds) of salt in tank  $T_i$  at time  $t$  (in minutes). Derive a system of linear differential equations for the  $y_i$ , including initial conditions.

**Problem 7.**

- (a) What is the period and the amplitude of  $3\cos(7t) - 5\sin(7t)$ ?
- (b) Assume that the angle  $\theta(t)$  of a swinging pendulum is described by  $\theta'' + 4\theta = 0$ . Suppose  $\theta(0) = \frac{3}{10}$  and  $\theta'(0) = -\frac{4}{5}$ . What is the period and the amplitude of the resulting oscillations?
- (c) The position  $y(t)$  of a certain mass on a spring is described by  $y'' + dy' + 5y = 0$ . For which value of  $d > 0$  is the system underdamped? Critically damped? Overdamped?
- (d) A forced mechanical oscillator is described by  $y'' + 2y' + y = 25\cos(2t)$ . As  $t \rightarrow \infty$ , what is the period and the amplitude of the resulting oscillations?
- (e) The motion of a certain mass on a spring is described by  $y'' + y' + \frac{1}{2}y = 5\sin(t)$  with  $y(0) = 2$  and  $y'(0) = 0$ . Determine  $y(t)$ . As  $t \rightarrow \infty$ , what are the period and amplitude of the oscillations?

**Problem 8.**

- (a) Consider the differential equation  $x^2y'' - 4xy' + 6y = 0$ . Find all solutions of the form  $y(x) = x^r$ .
- (b) Determine the general solution of  $x^2y'' - 4xy' + 6y = x^3$ .