

# Midterm #1

*Please print your name:*

---

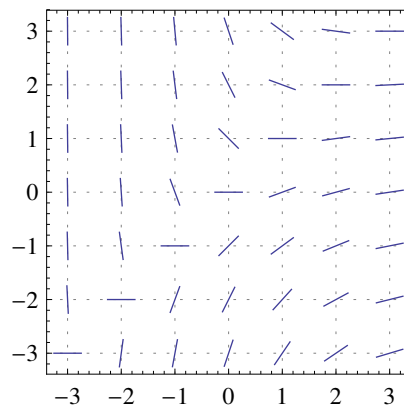
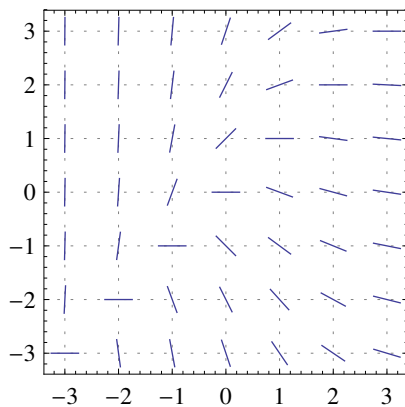
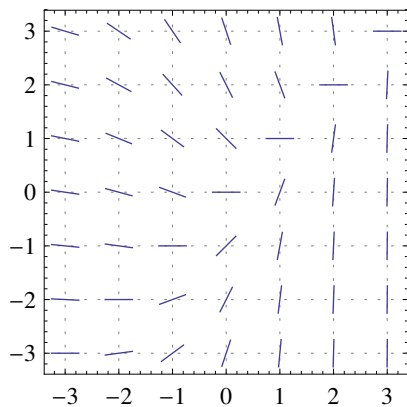
No notes, calculators or tools of any kind are permitted. There are 39 points in total. You need to show work to receive full credit.

**Good luck!**

**Problem 1. (9 points)** A tank contains 10gal of pure water. It is filled with brine (containing 3lb/gal salt) at a rate of 2gal/min. At the same time, well-mixed solution flows out at a rate of 1gal/min. How much salt is in the tank after  $t$  minutes?

After  $t$  minutes, the tank contains  $x(t)$  lb of salt, where  $x(t) =$

**Problem 2. (2 points)** Circle the slope field below which belongs to the differential equation  $e^x y' = x - y$ .



**Problem 3. (3 points)** Consider the initial value problem

$$(x + 2)y' = \sin(x + 3y), \quad y(a) = b.$$

For which values of  $a$  and  $b$  can we guarantee existence and uniqueness of a (local) solution?

The IVP has a unique local solution if

**Problem 4. (4 points)** Find the general solution to the differential equation  $y'' + y' = 2y$ .

$y(x) =$

**Problem 5. (7 points)** Determine the general solution to  $xy' = y + 4x^2 e^{2x}$ .

$y(x) =$

**Problem 6. (2 points)** A rising population is modeled by the equation  $\frac{dP}{dt} = 1000P - 10P^2$ . When the population size stabilizes in the long term, how big will the population be? Do not solve the DE!

**Problem 7. (6 points)** The time rate of change of a rabbit population  $P$  is proportional to the square root of  $P$ . At time  $t = 0$ , the population numbers 25 rabbits and is increasing at the rate of 5 rabbits per month. How many rabbits will there be after two months?

$$P(2 \text{ months}) =$$

**Problem 8. (6 points)** Solve the initial value problem  $x \frac{dy}{dx} = y + x e^{-y/x}$ ,  $y(1) = 0$ .

$$y(x) =$$

(extra scratch paper)