Midterm #1 – Practice

MATH 238 — Differential Equations I
Midterm: Thursday, Sept 29

Please print your name:

Reminder. No notes, calculators or tools of any kind will be permitted on the midterm exam.

Problem 1.

(a) Find the general solution to $y'' + y' = 12y$.

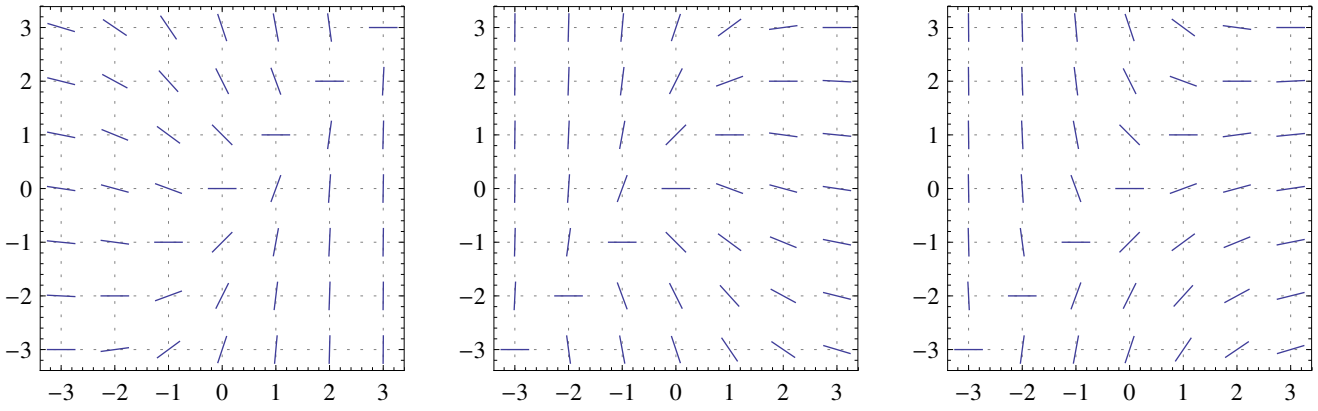
(b) Find the general solution to $y''' + y'' = 12y'$.

Problem 2. Consider the initial value problem

$$(xy + 2x)y' = \cos(x), \qquad y(a) = b.$$

For which values of a and b can we guarantee existence and uniqueness of a (local) solution?

Problem 3. Circle the slope field below which belongs to the differential equation $e^x y' = x - y$.



Problem 4. In the differential equation $x(y+1)\dfrac{dy}{dx} = (x^2+y)^3$ substitute $u = (x^2+y)^3$.

What is the resulting differential equation for u? No need to simplify!

Problem 5. Solve the initial value problem $y' = 2xy + 3x^2 e^{x^2}$, $y(0) = 5$.

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Problem 6. Find a general solution to the differential equation $x(x+y)y' = y(3x+y)$.

Problem 7. Find a general solution to the differential equation $\dfrac{\mathrm{d}y}{\mathrm{d}x} + y^2 \sin(x) = 0$.

Problem 8. Find a general solution to the differential equation $xy' = y + x^2\cos(x)$.

Problem 9. A tank contains 20gal of pure water. It is filled with brine (containing 5lb/gal salt) at a rate of 8gal/min. At the same time, well-mixed solution flows out at a rate of 6gal/min. How much salt is in the tank after t minutes?

Problem 10. The time rate of change of a rabbit population P is proportional to the square root of P. At time $t = 0$, the population numbers 100 rabbits and is increasing at the rate of 20 rabbits per month. How many rabbits will there be after two months?

Problem 11. A rising population is modeled by the differential equation $\dfrac{\mathrm{d}P}{\mathrm{d}t} = 1000P - 20P^2$.

(a) When the population size stabilizes in the long term, how big will the population be?

(b) Under which condition will the population size shrink?

(c) What is the population size when it is growing the fastest?

Problem 12. Solve the initial value problem $x\dfrac{\mathrm{d}y}{\mathrm{d}x} = y - xe^{y/x}, \quad y(1) = 0$.

Problem 13. Solve the initial value problem $(x^2+1)\dfrac{\mathrm{d}y}{\mathrm{d}x} + xy = \dfrac{1}{\sqrt{x^2+1}}, \quad y(0) = 1$.

Problem 14. Find a general solution to the differential equation $2 + \dfrac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{2x+y}$.

Problem 15. Find a general solution to the differential equation $x^2\dfrac{\mathrm{d}y}{\mathrm{d}x} - x^2 - y^2 - 3xy = 0$.

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