

Final: practice

Please print your name:

Bonus challenge. Let me know about any typos you spot in the posted solutions (or lecture sketches). Any typo, that is not yet fixed by the time you send it to me, is worth a bonus point.

Problem 1. The final exam will be comprehensive, that is, it will cover the material of the whole semester.

- (a) Do the practice problems for both midterms.
- (b) Retake both midterm exams.
- (c) Do the problems below. (Solutions are posted.)

Problem 2. The position $y(t)$ of a certain mass on a spring is described by $2y'' + dy' + 3y = F \sin(4\omega t)$.

- (a) Assume first that there is no external force, i.e. $F = 0$. For which values of d is the system overdamped?
- (b) Now, $F \neq 0$ and the system is undamped, i.e. $d = 0$. For which values of ω , if any, does resonance occur?

Problem 3.

- (a) Determine the Laplace transform $\mathcal{L}(2e^{4t} - 6e^{-t} + 3)$.
- (b) Determine the Laplace transform $\mathcal{L}((2t - 5)e^{-3t})$.
- (c) Determine the inverse Laplace transform $\mathcal{L}^{-1}\left(\frac{s - 16}{s^2 - 2s - 8}\right)$.

Problem 4. Determine the Laplace transform of the unique solutions. [Do not determine the solution and do not simplify.]

- (a) $y'' + 4y' - 3y = 2e^{-4t} + 5t^2$, $y(0) = 7$, $y'(0) = -2$.
- (b) $y'_1 = 3y_1 - 4y_2 - 5t^2e^{-3t}$, $y'_2 = y_1 + 2y_2$, $y_1(0) = 3$, $y_2(0) = -2$.

For the final exam, you will be provided the following table of Laplace transforms.

$f(t)$	$F(s)$
$f'(t)$	$sF(s) - f(0)$
$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
e^{at}	$\frac{1}{s - a}$
$\cos(\omega t)$	$\frac{s}{s^2 + \omega^2}$
$\sin(\omega t)$	$\frac{\omega}{s^2 + \omega^2}$
t^n	$\frac{n!}{s^{n+1}}$
$e^{at}f(t)$	$F(s - a)$
$tf(t)$	$-F'(s)$
$u_a(t)f(t - a)$	$e^{-sa}F(s)$