

Content Scramble System

CSS

used for encryption of DVDs
combines 2 LFSRs (nonlinearly!)

introduced 1996
broken 1999

baby CSS

$$\begin{aligned} \text{LFSR-1} & \quad X_{n+3} \equiv X_{n+1} + X_n \pmod{2} \\ \text{LFSR-2} & \quad X_{n+4} \equiv X_{n+2} + X_n \pmod{2} \end{aligned}$$

CSS-PRG: add outputs from LFSRs with carry (nonlinear)

EG

(0,0,1)
(0,1,0,1)

seeds for LFSRs

	seed	output
LFSR-1	0 0 1	0 1 1 1 0 0 1 0 ...
+	LFSR-2 0 1 0 1	0 0 0 1 0 1 0 0 ...
	carry	1
=	CSS-PRG	0 1 1 0 1 1 1 0 ...

→ repeats after 2^3-1 bits

comments

• less predictable than single LFSR

initial output 0 1 ... could have come from

$$\begin{aligned} & \begin{matrix} 01... \\ + 00... \end{matrix} \quad + \quad \begin{matrix} 10... \\ + 10... \end{matrix} \quad + \quad \begin{matrix} 11... \\ + 11... \end{matrix} \quad + \quad \begin{matrix} 00... \\ + 01... \end{matrix} \end{aligned}$$

don't learn about states of LFSRs but about their correlation

• the carry is crucial

$$\begin{aligned} & \text{LFSR-1} \quad a_1 \quad a_2 \quad \dots \\ + & \text{LFSR-2} \quad b_1 \quad b_2 \quad \dots \end{aligned}$$

without carry $a_1+b_1 \quad a_2+b_2 \quad \dots$ ← nonlinear! (addition mod 2)

with carry $a_1+b_1 \quad a_2+b_2+a_1b_1 \quad \dots$ (mod 2)

$$\left. \begin{matrix} \} \\ \text{carry} \end{matrix} \right\} \Leftrightarrow \begin{matrix} a_1=1 \\ \text{and } b_1=1 \end{matrix} \Leftrightarrow a_1b_1 \equiv 1 \pmod{2}$$