

Modern ciphers

Example 40. For modern ciphers, we will change the alphabet from A, B, \dots, Z to $0, 1$. One of the most common ways of encoding text is **ASCII**.

In ASCII (American Standard Code for Information Interchange), each letter is represented using 8 bits (1 byte).

Among the $2^8 = 256$ many characters are the usual letters, as well as common symbols.

For instance: $\text{space} = (20)_{16}$, $\text{"0"} = (30)_{16}$, $A = (41)_{16} = (0100, 0001)_2 = 65$, $a = (61)_{16} = (0110, 0001)_2 = 97$

See, for instance, <http://www.asciitable.com> for the full table.

Example 41. The new (8/2018) insignia of **FinCEN** feature binary digits. What do they mean?

01000110 01101001 01101110 01000011 01000101 01001110 <https://www.fincen.gov>

By the way. If you ever have more than \$10,000 in foreign accounts, you must file a report to FinCEN.

One-time pad

Definition 42. The “exclusive or” (XOR), often written \oplus , is defined bitwise:

	0	0	1	1
\oplus	0	1	0	1
$=$	0	1	1	0

Note. On the level of individual bits, this is just addition modulo 2.

By the way. Best thing about a boolean: even if you are wrong, you are only off by a bit.

Example 43. $1011 \oplus 1111 = 0100$

Example 44. Observe that $a \oplus b \oplus b = a$.

One way to see that is think bitwise in terms of addition modulo 2. Then, $a + b + b = a + 2b \equiv a \pmod{2}$.

A **one-time pad** works as follows. We use a key k of the same length as the message m . Then the ciphertext is

$$c = E_k(m) = m \oplus k.$$

To decipher, we use $m = D_k(c) = c \oplus k$.

As the name indicates, we must never use this key again!

Note. Observe that encryption and decryption are the same routine.

Comment. If that is helpful, a one-time pad is doing exactly the same as the Vigenere cipher if we use a key of the same length as the message (also, we use $0, 1$ as our letters instead of the classical A, B, \dots, Z).

Example 45. Using a one-time pad with key $k = 1100, 0011$, what is the message $m = 1010, 1010$ encrypted to?

Solution. $c = m \oplus k = 0110, 1001$

If a one-time pad (with perfectly random key) is used exactly once to encrypt a message, then **perfect confidentiality** is achieved (eavesdropping is hopeless).

Meaning that Eve intercepting the ciphertext can draw absolutely no conclusions about the plaintext (because, without information on the key, every text of the right length is actually possible and equally likely), see next example.

Example 46. A ciphertext only attack on the one-time pad is entirely hopeless. Explain why!

Solution. The attacker only knows $c = m \oplus k$. The attacker is unable to get any information on m , because every other message m' (of the right length) could have resulted in the same ciphertext c . Indeed, the key $k' = m' \oplus c$ encrypts m' to c as well (because $m' \oplus k' = m' \oplus (m' \oplus c) = c$). Moreover, every plaintext m' is equally likely because it corresponds to a unique key.

The next example highlights the importance of only using the key once.

Example 47. (attack on the two-time pad) Alice made a mistake and encrypted the two plaintexts m_1, m_2 using the same key k . How can Eve exploit that?

Solution. Eve knows the two ciphertexts $c_1 = m_1 \oplus k$ and $c_2 = m_2 \oplus k$.

Hence, she can compute $c_1 \oplus c_2 = (m_1 \oplus k) \oplus (m_2 \oplus k) = m_1 \oplus m_2$.

This means that Eve knows $m_1 \oplus m_2$, which is information about the original plaintexts (no key involved!). That's a cryptographic disaster: Eve should never be able to learn *anything* about the plaintexts.

In fact. If the plaintexts are, say, English text encoded using ASCII then Eve very possibly can (almost) reconstruct both m_1 and m_2 from $m_1 \oplus m_2$. The reason for that is that the messages are expressed in ASCII, which means 8 bits per character of text. However, the **entropy** (a measure for the amount of information in a message) of (longer) typical English text is frequently below 2 bits per character.

Some details and beautiful graphical illustration are given at:

<http://crypto.stackexchange.com/questions/59/taking-advantage-of-one-time-pad-key-reuse>

We saw in Example 46 that ciphertext only attacks on the one-time pad are entirely hopeless. What about other attacks?

Attacks like known plaintext or chosen plaintext don't apply if the key is only to be used once.

Yet, the one-time pad by itself provides **little protection of integrity**. The next example shows how tampering is possible without knowledge about the key.

Example 48. Alice sends an email to Bob using a one-time pad. Eve knows that and concludes that, per email standard, the plaintext must begin with To: Bob. Eve wants to tamper with the message and change it to To: Boo, for a light scare.

- Eve wants to change the 7th letter of the plain text m from b to o .
- Since b is $0x62$ and o is $0x6F$, we have $b \oplus o = 0x0D$. Hence, $b \oplus 0x0D = o$.
- Therefore, if $e = 0x\underbrace{000000000000D00\dots}_{6 \text{ characters}}$, then $\underbrace{\text{“TO: Bob...”}}_m \oplus e = \underbrace{\text{“TO: Boo...”}}_{m'}$.
- Alice sends $c = m \oplus k$. If Eve changes the ciphertext c to $c' = c \oplus e$, then Bob receives c' and decrypts it to $c' \oplus k = \underbrace{m \oplus k}_{=c} \oplus e \oplus k = m \oplus e = m'$, which is what Eve intended.

Using the one-time pad presents several challenges, including:

- keys must not be reused (see Example 47)
- while perfectly protecting against eavesdropping (if done correctly), the one-time pad is not secure against tampering (see Example 48)
- key distribution and management
 - Alice and Bob have to somehow exchange huge amounts of keys, so that, at a later time, they are able to communicate securely.
- for perfect confidentiality, the key must be perfectly random
 - But how can we produce huge amounts of random bits?
 - Especially, how to teach a deterministic machine like a computer to do that? Think about it! This is much more challenging than it may seem at first...

These issues make one-time pads difficult to use in practice.

Historic comment. During the Cold War, the “hot line” between Washington and Moscow apparently used one-time pads for secure communication.