Sketch of Lecture 1

Mon, 1/11/2016

Our course website is: `http://math227.straub.link`

These lecture sketches, exam preparation problems, solutions and other course material will be posted there. All scores and grades, on the other hand, are posted to Sakai.

Review

Example 1. $\int_0^1 (1-x^2)\mathrm{d}x =$

Solution. $\int_0^1 (1-x^2)\mathrm{d}x = \left[x - \frac{x^3}{3}\right]_0^1 = \frac{2}{3}$

Comment. Interpret the integral as computing an area under the curve $1-x^2$. From your (rough) sketch read off that the value of the integral has to be between $\frac{1}{2}$ and 1.

Polar coordinates and Euler's identity

Example 2. Draw a right-angled triangle with hypotenuse 1. Let θ be one of the other two angles. Express the lengths of the other sides in terms of trig functions.

The **polar coordinates** (r,θ) represent the point with **cartesian coordinates** $(x,y) = r(\cos\theta, \sin\theta)$.

Often, θ is taken from $[0,2\pi)$ (but $(-\pi,\pi]$ is another popular choice), and, usually, $r \geqslant 0$.

Example 3. Which point (in cartesian coordinates) has polar coordinates $r=3$, $\theta = \frac{\pi}{2}$?

Solution. $(x,y) = (0,3)$

Note. The polar coordinates $r=3$, $\theta = \frac{\pi}{2} + 2\pi$ correspond to the same point. Polar coordinates are not quite unique.

Example 4. Find polar coordinates for the point with cartesian coordinates $(-1,1)$.

Solution. The polar coordinates of $(-1,1)$ are $r=\sqrt{2}$ and $\theta = \frac{3\pi}{4}$.

Example 5. $\int \sin(x)\mathrm{d}x = -\cos(x) + C$, $\int \sin^2(x)\mathrm{d}x = ?$

Example 6. $\sin^2(\theta) = \frac{1-\cos(2\theta)}{2}$

Note. These sorts of trig identities are frequently useful. One can either remember them or, even better, understand how they follow from Euler's identity (below). For instance, the present identity is a consequence of $(e^x)^2 = e^{2x}$. [Note how, again, one side has the function squared and the other $2x$ in place of x.]

Theorem 7. (Euler's identity) $e^{i\theta} = \cos(\theta) + i\sin(\theta)$

A little more on Euler's identity (and how it explains trig identities) next time...

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