Quiz #8

MATH 126-102 — Calculus II
Thursday, Nov 5, 2026

Please print your name:

Problem 1. (10 points) Determine the value of the following series or state that they diverge.
(Show your work where necessary!)

(a) $\displaystyle\sum_{n=0}^{\infty}\left(\frac{4}{5}\right)^n =$

(b) $\displaystyle\sum_{n=2}^{\infty} 2^{-n} =$

(c) $\displaystyle\sum_{n=0}^{\infty}\frac{3^n-2^n}{4^n} =$

(d) $\displaystyle\sum_{n=0}^{\infty}\frac{3^n-5^n}{4^n} =$

(e) $\displaystyle\sum_{n=0}^{\infty}\frac{7n^2+3n}{5n^2+1} =$

Solution.

(a) $\displaystyle\sum_{n=0}^{\infty}\left(\frac{4}{5}\right)^n = \frac{1}{1-\frac{4}{5}} = 5$

(b) $\displaystyle\sum_{n=2}^{\infty} 2^{-n} = \frac{1}{1-\frac{1}{2}} - (2^{-0}+2^{-1}) = 2-\left(1+\frac{1}{2}\right) = \frac{1}{2}$

(c) $\displaystyle\sum_{n=0}^{\infty}\frac{3^n-2^n}{4^n} = \sum_{n=0}^{\infty}\left(\frac{3}{4}\right)^n - \sum_{n=0}^{\infty}\left(\frac{2}{4}\right)^n = \frac{1}{1-\frac{3}{4}} - \frac{1}{1-\frac{1}{2}} = 4-2 = 2$

(d) $\displaystyle\sum_{n=0}^{\infty}\frac{3^n-5^n}{4^n}$ diverges because $\displaystyle\frac{3^n-5^n}{4^n} \to \infty$ (this limit would have to be 0 for the series to converge) as $n \to \infty$.
(If you don't notice right away and proceed as in the previous item, you run into $\sum_{n=0}^{\infty}(\frac{5}{4})^n$ for which $\left|\frac{5}{4}\right| \not< 1$.)

(e) $\displaystyle\sum_{n=0}^{\infty}\frac{7n^2+3n}{5n^2+1}$ diverges because $\displaystyle\frac{7n^2+3n}{5n^2+1} \to \frac{7}{5} \neq 0$ as $n \to \infty$. □

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