Quiz #8

MATH 126-101 — Calculus II
Thursday, Nov 5, 2026

Please print your name:

Problem 1. (10 points) Determine the value of the following series or state that they diverge.
(Show your work where necessary!)

(a) $\displaystyle\sum_{n=0}^{\infty}\left(\frac{3}{4}\right)^n =$

(b) $\displaystyle\sum_{n=2}^{\infty} 3^{-n} =$

(c) $\displaystyle\sum_{n=0}^{\infty}\frac{2^n-3^n}{5^n} =$

(d) $\displaystyle\sum_{n=0}^{\infty}\frac{2^n+4^n}{3^n} =$

(e) $\displaystyle\sum_{n=0}^{\infty}\frac{5n^2+2n}{3n^2+1} =$

Solution.

(a) $\displaystyle\sum_{n=0}^{\infty}\left(\frac{3}{4}\right)^n = \frac{1}{1-\frac{3}{4}} = 4$

(b) $\displaystyle\sum_{n=2}^{\infty} 3^{-n} = \frac{1}{1-\frac{1}{3}} - (3^{-0}+3^{-1}) = \frac{3}{2} - \left(1+\frac{1}{3}\right) = \frac{1}{6}$

(c) $\displaystyle\sum_{n=0}^{\infty}\frac{2^n-3^n}{5^n} = \sum_{n=0}^{\infty}\left(\frac{2}{5}\right)^n - \sum_{n=0}^{\infty}\left(\frac{3}{5}\right)^n = \frac{1}{1-\frac{2}{5}} - \frac{1}{1-\frac{3}{5}} = \frac{5}{3} - \frac{5}{2} = -\frac{5}{6}$

(d) $\displaystyle\sum_{n=0}^{\infty}\frac{2^n+4^n}{3^n}$ diverges because $\displaystyle\frac{2^n+4^n}{3^n} \to \infty$ (this limit would have to be 0 for the series to converge) as $n \to \infty$.
(If you don't notice right away and proceed as in the previous item, you run into $\sum_{n=0}^{\infty}(\frac{4}{3})^n$ for which $\left|\frac{4}{3}\right| \not< 1$.)

(e) $\displaystyle\sum_{n=0}^{\infty}\frac{5n^2+2n}{3n^2+1}$ diverges because $\displaystyle\frac{5n^2+2n}{3n^2+1} \to \frac{5}{3} \neq 0$ as $n \to \infty$. □

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