Midterm #2

MATH 126-102 — Calculus II
Thursday, Oct 22, 2026

Please print your name:

No notes, graphing calculators or other tools are permitted. There are 30 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (6 points) Evaluate the following integral or show that it diverges: $\int_1^2 x^2 \ln(x)\,\mathrm{d}x$

Solution. We apply integration by parts, and use $\int f(x)g'(x)\mathrm{d}x = f(x)g(x) - \int f'(x)g(x)\mathrm{d}x$ with $f(x)=\ln(x)$ and $g'(x)=x^2$. With $g(x)=\frac{1}{3}x^3$, we then get

$$\int_1^2 x^2 \ln(x)\,\mathrm{d}x = \left[\frac{1}{3}x^3\ln(x)\right]_1^2 - \int_1^2 \frac{1}{x}\cdot\frac{1}{3}x^3\,\mathrm{d}x = \frac{8}{3}\ln(2) - \frac{1}{3}\int_1^2 x^2\,\mathrm{d}x = \frac{8}{3}\ln(2) - \frac{1}{3}\left[\frac{1}{3}x^3\right]_1^2 = \frac{8}{3}\ln(2) - \frac{7}{9}.$$

Problem 2. (3 points) Evaluate the following integral or show that it diverges: $\int_{-2}^2 \frac{1}{3x+1}\,\mathrm{d}x$

Solution. Note that the integrand has a singularity at $x=-\frac{1}{3}$. The antiderivative is

$$\int \frac{1}{3x+1}\,\mathrm{d}x = \frac{1}{3}\ln|3x+1| + C,$$

and we can already see that we will not get a finite contribution for $x=-\frac{1}{3}$ since $\lim_{x\to -1/3}\frac{1}{3}\ln|3x+1| = -\infty$. This means that our integral diverges.

Armin Straub
straub@southalabama.edu

Problem 3. (7 points) Evaluate the following indefinite integral: $\displaystyle\int \frac{x^2+1}{(x-1)(x+2)(x+3)}\,\mathrm{d}x$

Solution. We don't need to do long division first because the numerator is of degree lower than the denominator.

Partial fractions tells us that $\dfrac{x^2+1}{(x-1)(x+2)(x+3)} = \dfrac{A}{x-1} + \dfrac{B}{x+2} + \dfrac{C}{x+3}$ for certain numbers A, B, C.

- We multiply both sides with $(x-1)(x+2)(x+3)$ to clear denominators:

$$x^2+1 = (x+2)(x+3)A + (x-1)(x+3)B + (x-1)(x+2)C$$

- Set $x=1$ to get $2=12A$ so that $A=\frac{1}{6}$.

  Set $x=-2$ to get $5=-3B$ so that $B=-\frac{5}{3}$.

  Set $x=-3$ to get $10=4C$ so that $C=\frac{5}{2}$.

Hence: $\displaystyle\int \frac{x^2+1}{(x-1)(x+2)(x+3)}\,\mathrm{d}x = \frac{1}{6}\int\frac{\mathrm{d}x}{x-1} - \frac{5}{3}\int\frac{\mathrm{d}x}{x+2} + \frac{5}{2}\int\frac{\mathrm{d}x}{x+3} = \frac{1}{6}\ln|x-1| - \frac{5}{3}\ln|x+2| + \frac{5}{2}\ln|x+3| + C$

Problem 4. (4 points) Determine the shape (but not the exact numbers involved) of the partial fraction decompositions of:

(a) $\dfrac{2x^3-7x+1}{(x-1)^3(x+4)} =$ ☐

(b) $\dfrac{3x^5+2x}{(x^2+3)(x+2)} =$ ☐

Solution.

(a) $\dfrac{2x^3-7x+1}{(x-1)^3(x+4)} = \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{(x-1)^3} + \dfrac{D}{x+4}$

(b) Since the numerator degree is 5 but the denominator degree is 3, we have to do long division first. This will result in Ax^2+Bx+C plus a remainder. Overall:

$$\frac{3x^5+2x}{(x^2+3)(x+2)} = Ax^2+Bx+C+\frac{Dx+E}{x^2+3}+\frac{F}{x+2}$$

Armin Straub
straub@southalabama.edu

Problem 5. (6 points) Evaluate the following indefinite integral: $\int \sin^3(3x)\cos^6(3x)\,\mathrm{d}x$

Solution. We substitute $u=\cos(3x)$ since the corresponding sine term occurs with an odd exponent. Then $\frac{\mathrm{d}u}{\mathrm{d}x}=-3\sin(3x)$ or, equivalently, $\sin(3x)\mathrm{d}x=-\frac{1}{3}\mathrm{d}u$, so that we get

$$\begin{aligned}\int \sin^3(3x)\cos^6(3x)\,\mathrm{d}x &= -\frac{1}{3}\int \sin^2(3x)u^6\,\mathrm{d}u=-\frac{1}{3}\int (1-\cos^2(3x))u^6\,\mathrm{d}u=-\frac{1}{3}\int (1-u^2)u^6\mathrm{d}u\\ &= -\frac{1}{3}\left(\frac{u^7}{7}-\frac{u^9}{9}\right)+C=\frac{\cos^9(3x)}{27}-\frac{\cos^7(3x)}{21}+C.\end{aligned}$$

Problem 6. (4 points) Evaluate the following indefinite integral: $\displaystyle\int \frac{1}{\sqrt{4-x^2}}\,\mathrm{d}x.$

Solution. We substitute $x=2\sin\theta$ because then $4-x^2=4(1-\sin^2\theta)=4\cos^2\theta$. Since $\frac{\mathrm{d}x}{\mathrm{d}\theta}=2\cos\theta$, we get

$$\int \frac{1}{\sqrt{4-x^2}}\,\mathrm{d}x=\int \frac{1}{\sqrt{4\cos^2\theta}}\,2\cos\theta\mathrm{d}\theta=\int 1\,\mathrm{d}\theta=\theta+C=\arcsin\left(\frac{x}{2}\right)+C.$$

Armin Straub
straub@southalabama.edu

(extra scratch paper)