Quiz #9

MATH 126 — Calculus II
Thursday, Nov 14

Please print your name:

Problem 1. (3 points) Write down the geometric series. When does it converge, and what does it converge to?

Solution. The geometric series $\sum_{n=0}^{\infty} x^n$ converges if and only if $|x| < 1$. In that case, $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$.

Problem 2. (2 points) Write down the p-series. When does it converge?

Solution. The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if and only if $p > 1$.

Problem 3. (5 points) Determine the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{3^n}{n} (x-2)^n$.

Solution. We apply the ratio test with $a_n = \frac{3^n}{n}(x-2)^n$.

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{3^{n+1}(x-2)^{n+1}}{n+1} \frac{n}{3^n(x-2)^n}\right| = 3|x-2| \frac{n}{n+1} \to 3|x-2| \text{ as } n \to \infty$$

The ratio test implies that $\sum_{n=1}^{\infty} \frac{3^n}{n}(x-2)^n$ converges if $|x-2| < \frac{1}{3}$.

Therefore, the radius of convergence is $\frac{1}{3}$. □

Problem 4. (bonus!) What is the exact interval of convergence for the series above?

Solution. The ratio test is inconclusive for $|x-2| = \frac{1}{3}$ or, equivalently, $x = 2 + \frac{1}{3} = \frac{7}{3}$ and $x = 2 - \frac{1}{3} = \frac{5}{3}$:

- $x = \frac{7}{3}$: $\sum_{n=1}^{\infty} \frac{3^n}{n}\left(\frac{1}{3}\right)^n = \sum_{n=1}^{\infty} \frac{1}{n}$ is the harmonic series (p-series with $p = 1$) which diverges (because $p \leqslant 1$).
- $x = \frac{5}{3}$: $\sum_{n=1}^{\infty} \frac{3^n}{n}\left(-\frac{1}{3}\right)^n = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges by the alternating series test ($\lim_{n\to\infty} \frac{1}{n} = 0$).

Hence, the exact interval of convergence is $\left[\frac{5}{3}, \frac{7}{3}\right)$. □

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