Quiz #8

MATH 126 — Calculus II
Thursday, Nov 7

Please print your name:

Problem 1. (10 points) Determine the value of the following series or state that they diverge.
(Show your work where necessary!)

(a) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n =$

(b) $\displaystyle\sum_{n=2}^{\infty} 2^{-n} =$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n - 3^n}{5^n} =$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n - 5^n}{3^n} =$

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{2n^2 - 3n}{5n^2 + 1} =$

Solution.

(a) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n = \frac{1}{1 - \frac{2}{3}} = 3$

(b) $\displaystyle\sum_{n=2}^{\infty} 2^{-n} = \frac{1}{1 - \frac{1}{2}} - (2^{-0} + 2^{-1}) = 2 - \left(1 + \frac{1}{2}\right) = \frac{1}{2}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n - 3^n}{5^n} = \sum_{n=0}^{\infty} \left(\frac{2}{5}\right)^n - \sum_{n=0}^{\infty} \left(\frac{3}{5}\right)^n = \frac{1}{1 - \frac{2}{5}} - \frac{1}{1 - \frac{3}{5}} = \frac{5}{3} - \frac{5}{2} = -\frac{5}{6}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n - 5^n}{3^n}$ diverges because $\displaystyle\frac{2^n - 5^n}{3^n} \to \infty$ (this limit would have to be 0 for the series to converge) as $n \to \infty$.

(If you don't notice right away and proceed as in the previous item, you run into $\sum_{n=0}^{\infty} \left(\frac{5}{3}\right)^n$ for which $\left|\frac{5}{3}\right| \not< 1$.)

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{2n^2 - 3n}{5n^2 + 1}$ diverges because $\displaystyle\frac{2n^2 - 3n}{5n^2 + 1} \to \frac{2}{5} \neq 0$ as $n \to \infty$. $\square$

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