

# Quiz #7

Please print your name:

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**Problem 1.** (1+2+2+2 points) Determine the following limits.

(No need to show work!)

(a)  $\lim_{n \rightarrow \infty} \frac{2}{n} =$

(b)  $\lim_{n \rightarrow \infty} \frac{7n^2 - 8n + 1}{2n^2 + 3} =$

(c)  $\lim_{n \rightarrow \infty} \sqrt{\frac{2n}{n+1}} =$

(d)  $\lim_{n \rightarrow \infty} \sin\left(\frac{\pi}{2} + \frac{1}{n}\right) =$

**Solution.**

(a)  $\lim_{n \rightarrow \infty} \frac{2}{n} = 0$

(b)  $\lim_{n \rightarrow \infty} \frac{7n^2 - 8n + 1}{2n^2 + 3} = \frac{7}{2}$

(c)  $\lim_{n \rightarrow \infty} \sqrt{\frac{2n}{n+1}} = \sqrt{2}$

(d)  $\lim_{n \rightarrow \infty} \sin\left(\frac{\pi}{2} + \frac{1}{n}\right) = \sin\left(\frac{\pi}{2}\right) = 1$

□

**Problem 2.** (3 points) Determine the following limit:  $\lim_{n \rightarrow \infty} \sqrt[n]{n^3}$

(Make sure to show all your work!)

**Solution.**  $\lim_{n \rightarrow \infty} \sqrt[n]{n^3} = \lim_{n \rightarrow \infty} n^{3/n} = \lim_{n \rightarrow \infty} \exp(\ln(n^{3/n})) = \lim_{n \rightarrow \infty} \exp\left(\frac{3}{n} \ln(n)\right) = \exp(0) = 1$

Here, we used that  $\lim_{n \rightarrow \infty} \frac{\ln n}{n} = 0$  (the logarithm grows slower than any power of  $n$ ). At this point, we can just use this fact. If that is not clear to us, we could use L'Hospital (since the limit is of the form  $\frac{\infty}{\infty}$ ):  $\lim_{n \rightarrow \infty} \frac{\ln n}{n} \stackrel{\text{LH}}{=} \lim_{n \rightarrow \infty} \frac{1/n}{1} = 0$ .

□