Midterm #3 – Practice

MATH 126 — Calculus II
Midterm: Thursday, Nov 21, 2024

Please print your name:

Bonus challenge. Let me know about any typos you spot in the posted solutions (or lecture sketches). Any mathematical typo, that is not yet fixed by the time you send it to me, is worth a bonus point.

Reminder. A nongraphing calculator (equivalent to the TI-30XIIS) is allowed on the exam (but not needed). No notes or further tools of any kind will be permitted on the midterm exam.

Problem 1. Go over all the quizzes since the last midterm exam!

To help you with that, there is a version of each quiz posted on our course website without solutions (of course, there are solutions, too).

Problem 2. Determine the Taylor polynomial of order 3 for $f(x)=\sqrt{x+2}$ at $x=2$.

Problem 3. Determine the following limits or state that the limit does not exist.

(a) $\lim_{n\to\infty} \frac{1+\log(n)}{1+n^{3/2}}$

(b) $\lim_{n\to\infty} \sin\left(\frac{5+3n^2}{\sqrt{1+2n^4}}\right)$

(c) $\lim_{n\to\infty} \frac{2-7n^2}{n(1+5n^2)}$

(d) $\lim_{n\to\infty} \frac{(5n^2+1)^2}{3n^4+2n^3+7}$

(e) $\lim_{n\to\infty} \frac{5^n+3^n}{4^n-1}$

(f) $\lim_{n\to\infty} \frac{3-2\log(n)}{4+2\log(n)}$

(g) $\lim_{n\to\infty} \frac{3n-2\log(n)}{4n+2\log(n)}$

(h) $\lim_{n\to\infty} \sqrt[n]{3n^2+1}$

(i) $\lim_{n\to\infty} \left(1-\frac{1}{2n}\right)^n$

Problem 4. Determine whether the following series converge or diverge. In each case, indicate a reason.

(a) $\sum_{n=1}^{\infty} \frac{7\sqrt{n}+\log(n)}{n^2+4}$

(b) $\sum_{n=2}^{\infty} \frac{\sqrt{n}}{10\log(n)}$

(c) $\sum_{n=1}^{\infty} \frac{n-4\sqrt{n}}{n^2+\log(n)}$

(d) $\sum_{n=1}^{\infty} \frac{5n+2}{n^4+1}$

(e) $\sum_{n=2}^{\infty} \frac{1-3\sqrt{n}}{1+6\sqrt{n}}$

(f) $\sum_{n=0}^{\infty} \frac{(-4)^n}{7n^2+1}$

Problem 5. Determine whether the following series converge or diverge. In each case, indicate a reason.

If a series converges, further state whether it converges absolutely or only converges conditionally.

(a) $\sum_{n=1}^{\infty} (-1)^n\sqrt{n}$

(b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n^2+3}}$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+3}$

(d) $\sum_{n=0}^{\infty} \frac{n!}{4^n\sqrt{n^2+3}}$

(e) $\sum_{n=0}^{\infty} \frac{(-1)^n}{4^n\sqrt{n^2+3}}$

(f) $\sum_{n=0}^{\infty} \frac{2^n}{n!}$

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Problem 6. Compute the following series (or state that it diverges):

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{4^n - 6 \cdot 2^n}{5^n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{4 - 2^n}{3^n}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{4^n - 2}{3^n}$

Problem 7. For which values of x does $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n x^n + 1}{3^n}$ converge? Compute the series for these values.

Problem 8. Determine the radius of convergence of the following power series.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{n!(x+1)^n}{10^n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(5x-2)^n}{n 3^n}$ (c) $\displaystyle\sum_{n=0}^{\infty} \binom{2n}{n} x^n$

Recall that $\binom{2n}{n} = \frac{(2n)!}{n!n!}$.

Problem 9. Consider the power series $\displaystyle\sum_{n=1}^{\infty} \frac{n \cdot 3^n}{5^n} (x-2)^n$.

(a) Determine the radius of convergence R.

(b) What is the exact interval of convergence?

(c) Let $f(x) = \displaystyle\sum_{n=1}^{\infty} \frac{n \cdot 3^n}{5^n} (x-2)^n$ for x such that $|x-2| < R$. Write down a power series for $f'(x)$.

Problem 10. Consider the power series $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{\sqrt{n}} (x-1)^n$.

(a) Determine the radius of convergence R.

(b) What is the exact interval of convergence?

(c) Let $f(x) = \displaystyle\sum_{n=1}^{\infty} \frac{3^n}{\sqrt{n}} (x-1)^n$ for x such that $|x-1| < R$. Write down a power series for $f'(x)$.

Problem 11. Using the integral test, determine whether the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n (\log n)^{3/2}}$ converges.

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