Midterm #1

MATH 126 — Calculus II
Thursday, Sept 26, 2024

Please print your name:

No notes, graphing calculators or other tools are permitted. There are 30 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (3 points) Evaluate the following indefinite integrals.

(a) $\displaystyle\int \frac{\mathrm{d}x}{2x} =$

(b) $\displaystyle\int \sin(5x)\,\mathrm{d}x =$

(c) $\displaystyle\int \frac{\mathrm{d}x}{x^2+1} =$

Problem 2. (5 points) Using the shell method, set up an integral (but do not evaluate it) for the volume of the solid obtained by revolving about the y-axis the region (in the first quadrant) enclosed by the curves

$$y = \frac{1}{x}, \quad y = \frac{1}{x^2}, \quad x = 3.$$

Problem 3. (2 points) Set up an integral (but do not evaluate it) for the length of the curve $y = x^3$ for $1 \leqslant x \leqslant 2$.

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Problem 4. (5 points) Evaluate the following indefinite integral: $\int \cos(3t)\sin^5(3t)\mathrm{d}t$

Problem 5. (5 points) Evaluate the integral $\int_0^2 \frac{x^2}{\sqrt{x^3+1}}\,\mathrm{d}x$.

Problem 6. (5 points) Solve the initial value problem $\frac{\mathrm{d}y}{\mathrm{d}x} = y^2, \;\; y(0) = 1.$

Problem 7. (5 points) A conical container of radius 10 ft and height 20 ft is completely filled with water (the tip of the cone is at the bottom). Write down an integral for how much work it will take to pump the water to a level of 5 ft above the cone's rim. Do not evaluate the integral. [Water weighs 62.4 lb/ft^3.]

Problem 8. (tiny bonus!) Very roughly, what is the distance from us to the moon?

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(extra scratch paper)