

Midterm #1 – Practice

MATH 126 — Calculus II
Midterm: Thursday, Sept 26, 2024

Please print your name:

Bonus challenge. Let me know about any typos you spot in the posted solutions (or lecture sketches). Any mathematical typo, that is not yet fixed by the time you send it to me, is worth a bonus point.

Reminder. A nongraphing calculator (equivalent to the TI-30XIIS) is allowed on the exam (but not needed). No notes or further tools of any kind will be permitted on the midterm exam.

Problem 1. Go over all the quizzes!

To help you with that, there is a version of each quiz posted on our course website without solutions (of course, there are solutions, too).

Problem 2. Find the length of the following curve:

$$y = 1 - 2x^{3/2}, \quad 0 \leq x \leq \frac{1}{3}.$$

Problem 3.

(a) Evaluate the integral $\int_0^2 \frac{x}{\sqrt{4-x^2}} dx$.

(b) Evaluate the integral $\int_0^2 \frac{x^3}{\sqrt{4-x^2}} dx$.

Problem 4. Solve the initial value problem

$$\frac{dy}{dx} = \frac{y^2}{x^2 + 1}, \quad y(0) = 2.$$

Problem 5. Consider the region (in the first quadrant) enclosed by the curves

$$y = \frac{1}{x}, \quad y = \frac{1}{x^2}, \quad x = 2.$$

- Sketch the region and find its area.
- Find the volume of the solid obtained by revolving this region about the line $y = 0$.
- Find the volume of the solid obtained by revolving this region about the line $y = -1$.
- Using the shell method, find the volume of the solid obtained by revolving this region about the y -axis.
- Using the washer method, compute the same volume.

Problem 6. Evaluate the following indefinite integral: $\int x \cos(x^2 + 3) dx$.

Problem 7. Consider the region bounded by the curves

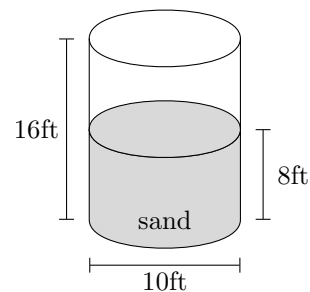
$$y = 0, \quad y = \sin(x), \quad 0 \leq x \leq \pi.$$

Sketch the region, then set up an integral (but do not evaluate it) for the volume of the solid obtained by rotating this region about the x -axis.

- (a) Set up an integral (but do not evaluate it) for the volume of the solid obtained by rotating this region about the x -axis.
- (b) Set up an integral (but do not evaluate it) for the volume of the solid obtained by revolving the region about the y -axis.

Problem 8. Consider the cylindrical container displayed to the right. It is half filled with sand weighing 100 lb/ft^3 .

- (a) Determine the amount of work needed to lift the sand to the rim of the tank.
- (b) Determine the amount of work needed to lift the sand to a level 10 ft above the rim of the tank. Just an integral is good enough, here.
- (c) Now, suppose the container is completely filled with sand. Determine the amount of work needed to lift the sand to a level 10 ft above the rim of the tank. Again, an integral is good enough, here.



Problem 9. A conical container of radius 15 ft and height 20 ft is completely filled with water (the tip of the cone is at the bottom). Write down an integral for how much work it will take to pump the water to a level of 10 ft above the cone's rim. Do not evaluate the integral.

[Water weighs 62.4 lb/ft^3 .]