Quiz #6

MATH 126 — Calculus II
Friday, Oct 16

Please print your name:

Problem 1. Determine the following limits.

(a) $\lim_{n\to\infty} \frac{2}{n} =$ ☐

(b) $\lim_{n\to\infty} \frac{7n^2-8n+1}{2n^2+3} =$ ☐

(c) $\lim_{n\to\infty} \sqrt{\frac{2n}{n+1}} =$ ☐

(d) $\lim_{n\to\infty} \sin\left(\frac{\pi}{2}+\frac{1}{n}\right) =$ ☐

Solution.

(a) $\lim_{n\to\infty} \frac{2}{n} = 0$

(b) $\lim_{n\to\infty} \frac{7n^2-8n+1}{2n^2+3} = \frac{7}{2}$

(c) $\lim_{n\to\infty} \sqrt{\frac{2n}{n+1}} = \sqrt{2}$

(d) $\lim_{n\to\infty} \sin\left(\frac{\pi}{2}+\frac{1}{n}\right) = \sin\left(\frac{\pi}{2}\right) = 1$ ☐

Problem 2. Determine the following limit: $\lim_{n\to\infty} \left(\frac{3}{n}\right)^{1/n}$ (Make sure to show all your work!)

Solution. If $\lim_{n\to\infty} \left(\frac{3}{n}\right)^{1/n} = L$, then $\lim_{n\to\infty} \log\left(\left(\frac{3}{n}\right)^{1/n}\right) = \log(L)$. We can compute the latter as

$$\lim_{n\to\infty} \log\left(\left(\frac{3}{n}\right)^{1/n}\right) = \lim_{n\to\infty} \frac{\log\left(\frac{3}{n}\right)}{n} = \lim_{n\to\infty} \frac{\log(3)-\log(n)}{n} \underset{\text{L'Hospital}}{\overset{\text{“}\frac{\infty}{\infty}\text{”}}{=}} \lim_{n\to\infty} \frac{-\frac{1}{n}}{1} = 0.$$

From $\log(L) = 0$ we conclude $L = e^0 = 1$. So, $\lim_{n\to\infty} \left(\frac{3}{n}\right)^{1/n} = 1$. ☐

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