Homework #10

MATH 126 — Calculus II
Friday, Nov 6

Please print your name:

Problem 1. (9.4.10) Does the series $\sum_{n=1}^{\infty} \sqrt{\frac{n+1}{n^2+2}}$ converge or diverge?

Solution. $\sum_{n=1}^{\infty} \sqrt{\frac{n+1}{n^2+2}}$ diverges. (Limit comparison with $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$, which diverges).

To give the details, let $a_n = \sqrt{\frac{n+1}{n^2+2}}$ and $b_n = \frac{1}{\sqrt{n}}$.

Then $\frac{a_n}{b_n} = \sqrt{\frac{n+1}{n^2+2}}\sqrt{n} = \sqrt{\frac{n^2+n}{n^2+2}} \to 1$ as $n \to \infty$. Hence, $\sum_{n=1}^{\infty} a_n$ converges if and only if $\sum_{n=1}^{\infty} b_n$ converges. Since it is a p-series with $p = \frac{1}{2} \leqslant 1$, we know that $\sum_{n=1}^{\infty} b_n$ diverges. Therefore, $\sum_{n=1}^{\infty} a_n$ diverges, too. □

Problem 2. (9.4.18) Does the series $\sum_{n=1}^{\infty} \frac{3}{n+\sqrt{n}}$ converge or diverge?

Solution. $\sum_{n=1}^{\infty} \frac{3}{n+\sqrt{n}}$ diverges. (Limit comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$, which diverges).

To give the details, let $a_n = \frac{3}{n+\sqrt{n}}$ and $b_n = \frac{1}{n}$.

Then $\frac{a_n}{b_n} = \frac{3n}{n+\sqrt{n}} \to 1$ as $n \to \infty$. Hence, $\sum_{n=1}^{\infty} a_n$ converges if and only if $\sum_{n=1}^{\infty} b_n$ converges. Since it is a p-series with $p = 1 \leqslant 1$, we know that $\sum_{n=1}^{\infty} b_n$ diverges. Therefore, $\sum_{n=1}^{\infty} a_n$ diverges, too. □

Problem 3. (9.5.18) Does the series $\sum_{n=1}^{\infty} n^2 e^{-n}$ converge or diverge?

Solution. $\sum_{n=1}^{\infty} n^2 e^{-n}$ converges by the ratio test.

Indeed, if $a_n = n^2 e^{-n}$, then $\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(n+1)^2 e^{-(n+1)}}{n^2 e^{-n}}\right| = \frac{(n+1)^2}{n^2} e^{-1} \to \frac{1}{e}$ as $n \to \infty$. Since $\frac{1}{e} < 1$, the series converges. □

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