Homework #2

MATH 126 — Calculus II
Friday, Sept 4

Please print your name:

Problem 1. (5.6.31) Evaluate the integral

$$\int_2^4 \frac{\mathrm{d}x}{x(\ln x)^2}.$$

Solution. We substitute $u = \ln x$. When $x = 2$, then $u = \ln(2)$, and when $x = 4$, then $u = \ln(4)$. Further using

$$\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x} \quad \Rightarrow \quad \frac{1}{x}\mathrm{d}x = \mathrm{d}u,$$

we therefore obtain

$$\int_2^4 \frac{\mathrm{d}x}{x(\ln x)^2} = \int_{\ln(2)}^{\ln(4)} \frac{1}{u^2}\mathrm{d}u = \left[-\frac{1}{u}\right]_{\ln(2)}^{\ln(4)} = \frac{1}{\ln(2)} - \frac{1}{\ln(4)}.$$

Using the fact that $\ln(4) = \ln(2^2) = 2\ln(2)$, this simplifies to $\frac{1}{\ln(2)} - \frac{1}{2\ln(2)} = \frac{1}{2\ln(2)} = \frac{1}{\ln(4)}$. □

Problem 2. (5.6.95) Find the area of the region in the first quadrant bounded by the line $y = x$, the line $x = 2$, the curve $y = 1/x^2$, and the x-axis.

Solution. Make a sketch! The sketch reveals that we need to find the intersection of $y = 1/x^2$ and $y = x$ (this will be the left-most point of our region).

$$\frac{1}{x^2} = x \quad \Rightarrow \quad 1 = x^3 \quad \Rightarrow \quad x = 1$$

For $0 \leqslant x \leqslant 1$, our region is bounded above by $y = x$ (and below by $y = 0$), while, for $1 \leqslant x \leqslant 2$, our region is bounded above by $y = 1/x^2$ (and below by $y = 0$). Hence, our area is

$$\int_0^1 x\,\mathrm{d}x + \int_1^2 \frac{1}{x^2}\mathrm{d}x = \left[\frac{1}{2}x^2\right]_0^1 + \left[-\frac{1}{x}\right]_1^2 = \frac{1}{2} + \left(-\frac{1}{2} + 1\right) = 1.$$

□

Problem 3. (6.1.42) Consider the region bounded by the curves $y = 4 - x^2$ and $y = 2 - x$. Find the volume of the solid generated by revolving this region about the x-axis.

Solution. Make a sketch! The sketch reveals that we need to find the two intersections of $y = 4 - x^2$ and $y = 2 - x$.

$$4 - x^2 = 2 - x \quad \Longleftrightarrow \quad x^2 - x - 2 = 0 \quad \Longleftrightarrow \quad x = -1 \text{ or } x = 2$$

Since the curve $y = 4 - x^2$ lies above $y = 2 - x$ for $-1 \leqslant x \leqslant 2$, our volume is

$$\int_{-1}^2 [\pi(4 - x^2)^2 - \pi(2 - x)^2]\mathrm{d}x.$$

To compute this integral, we multiply out $(4 - x^2)^2 = 16 - 8x^2 + x^4$ and $(2 - x)^2 = 4 - 4x + x^2$. Therefore,

$$\int_{-1}^2 [\pi(4 - x^2)^2 - \pi(2 - x)^2]\mathrm{d}x = \pi\int_{-1}^2 [x^4 - 9x^2 + 4x + 12]\mathrm{d}x = \pi\left[\frac{x^5}{5} - 3x^3 + 2x^2 + 12x\right]_{-1}^2 = \frac{108\pi}{5}.$$

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□

Problem 4. (6.1.52) Find the volume of the solid generated by revolving the triangular region bounded by the lines $y=2x$, $y=0$, and $x=1$ about

(a) the line $x=1$,

(b) the line $x=2$.

Solution. Make a sketch! The sketch reveals that our region extends from $y=0$ to $y=2$. Note that $x=y/2$.

(a) The volume is

$$\int_0^2 \pi\left(1-\frac{y}{2}\right)^2 \mathrm{d}y = \int_0^2 \pi\left(1-y+\frac{y^2}{4}\right) \mathrm{d}y = \pi\left[y-\frac{y^2}{2}+\frac{y^3}{12}\right]_0^2 = \frac{2\pi}{3}.$$

(b) The volume is

$$\int_0^2 \pi\left[\left(2-\frac{y}{2}\right)^2-(2-1)^2\right] \mathrm{d}y = \int_0^2 \pi\left[3-2y+\frac{y^2}{4}\right] \mathrm{d}y = \pi\left[3y-y^2+\frac{y^3}{12}\right]_0^2 = \frac{8\pi}{3}.$$

□

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