Preparing for the Final

MATH 126 — Calculus II
Wednesday, Dec 9

Please print your name:

Problem 1. Go over all past quizzes!

Problem 2. Study the practice problems for the two midterm exams!

Problem 3. Retake the two midterm exams!

(A copy without solutions is available on our course website. Of course, you also find solutions there.)

Additional problems covering the material since the second midterm

Problem 4.

(a) Write down the Taylor series for $\sin(2x)$ at $x=0$.

(b) What is the radius of convergence of that series? What is the exact interval of convergence?

Problem 5. Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{(x+1)^n}{\sqrt{n}\, 3^n}$.

(a) What is the radius of convergence of that series?

(b) What is the exact interval of convergence?

Problem 6. For each of the following series, decide whether it converges absolutely, converges conditionally, or diverges.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{n!}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \sqrt{n}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n^2+3}}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+3}$

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{n!}{4^n\sqrt{n^2+3}}$

(f) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{4^n\sqrt{n^2+3}}$

Problem 7. Let $f(x) = \dfrac{1}{(1-2x)^2}$.

(a) Determine $f'(x)$ as well as $\displaystyle\int f(x)\,\mathrm{d}x$.

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(b) Use your answer for $\int f(x)\,\mathrm{d}x$ to determine the Taylor series for $f(x)$ at $x=0$.

(c) What is the radius of convergence of that series?

(d) What is the exact interval of convergence?

Problem 8.

(a) Determine the Taylor series for $\int e^{-x^2/2}\,\mathrm{d}x$ at $x=0$.

(b) What is the radius of convergence of that series? What is the exact interval of convergence?

Problem 9.

(a) What are the cartesian coordinates of the point with polar coordinates $r=3$, $\theta=\frac{\pi}{6}$?

(b) What are the polar coordinates of the points with cartesian coordinates $(-2,2)$ and $(2,-2)$?

(c) Sketch the region described by $2\leqslant r\leqslant 4$, $-\frac{\pi}{2}\leqslant\theta\leqslant\frac{3\pi}{4}$.

Problem 10. Consider the parametric curve given by $x=t\cos(t)$, $y=t\sin(t)$ with parameter $t\in[0,2\pi]$.

(a) Make a very rough sketch of the curve.

(b) Find the slope of the line tangent to the curve at the point corresponding to $t=\frac{\pi}{2}$.

(c) Setup an integral for the arc length of the curve.

Simplify but don't evaluate the integral. [However, if you want a real challenge, evaluate it using our techniques.]

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