Bonus #1

MATH 126 — Calculus II
due before Friday, Sept 25

Please print your name:

Problem 1. Consider the region enclosed by the curves

$$y = \sqrt{x}, \quad y = 0, \quad x = 0, \quad x = 4.$$

We are interested in the solid that is obtained by revolving this region about the x-axis.

(a) Make a sketch of the region and the solid of revolution.

(b) For later comparison, compute the volume of this solid by slicing vertically (so that each slice looks like a little disk). Nothing new here, this is what we did in class.

(c) Now, we compute the (same) volume of this solid by operating horizontally.

- Sketch a small horizontal slice of our region. What does it look like when we revolve only this slice about the x-axis? (It should look like a *cylindrical shell*, see Section 6.2 for pictures.)
- Compute the volume of our solid by "adding" up the volumes of all these cylindrical shells. Setup the appropriate integral and then find its value. (You are encouraged to read through Section 6.2 for assistance.)
- Make sure that you get the same final volume as in (b).

Solution.

(a) Done in class. Too lazy to produce a sketch here...

(b) The volume is

$$\int_0^4 \pi(\sqrt{x})^2\,\mathrm{d}x = \pi\int_0^4 x\,\mathrm{d}x = \pi\left[\frac{1}{2}x^2\right]_0^4 = 8\pi.$$

(c) We operate between $y = 0$ and $y = \sqrt{4} = 2$. A slice at height y has width $4 - y^2$ (note that $y = \sqrt{x}$ implies $x = y^2$). Giving this slice a thickness of $\mathrm{d}y$ and revolving about the x-axis, we obtain a cylindrical shell with approximate volume

$$(\text{circumference}) \times (\text{width}) \times (\text{thickness}) = (2\pi y)(4 - y^2)\,\mathrm{d}y.$$

"Summing" all these volumes, we get

$$\int_0^2 2\pi y(4 - y^2)\,\mathrm{d}y = 2\pi\left[2y^2 - \frac{1}{4}y^4\right]_0^2 = 2\pi(8 - 4) = 8\pi.$$

Sure enough, the volume is exactly what we calculated before.

□

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