Worksheet #1

MATH 120 — Calculus & Applications
Thursday, Aug 22

Please print your name:

Problem 1. (warmup)

(a) Express each in the form x^r:

$$\sqrt{x} = \boxed{x^{1/2}}, \quad \frac{1}{x^2} = \boxed{x^{-2}}, \quad x^3 \cdot x^7 = \boxed{x^{10}}, \quad (x^3)^7 = \boxed{x^{21}}, \quad \frac{x^3}{x^7} = \boxed{x^{-4}}$$

(b) The equation $ax^2 + bx + c = 0$ has up to $\boxed{2}$ solutions given by $\boxed{\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$.

(c) If $f(x) = 2x - \frac{1}{x}$, then $f(3) = \boxed{6 - \frac{1}{3} = \frac{17}{3}}$ and $f\left(\frac{1}{x-1}\right) = \boxed{\frac{2}{x-1} - x + 1}$.

Problem 2. Factor:

(a) $x^2 - 9 = \boxed{(x-3)(x+3)}$

(b) $3x^2 - 12x - 36 = \boxed{3(x-6)(x+2)}$

(c) $5x^2 + 2x - 1 = \boxed{5\left(x + \frac{1+\sqrt{6}}{5}\right)\left(x + \frac{1-\sqrt{6}}{5}\right)}$

Solution. You can always use the abc-formula to factor quadratic polynomials (though you should be able to see the first factorization; and, if you have some practice, you can see the second one as well).

In the third case, $x = \frac{-2 \pm \sqrt{4+20}}{10} = \frac{-2 \pm \sqrt{24}}{10} = \frac{-1 \pm \sqrt{6}}{5}$.

Hence, $5x^2 + 2x - 1 = 5\left(x - \frac{-1-\sqrt{6}}{5}\right)\left(x - \frac{-1+\sqrt{6}}{5}\right)$, which simplifies to the above. □

Problem 3. Find the intercepts.

(a) $2x + 3y = 12$ — The x-intercepts are $\boxed{(6,0)}$, and the y-intercepts are $\boxed{(0,4)}$.

(b) $y = 7$ — The x-intercepts are $\boxed{\text{none}}$, and the y-intercepts are $\boxed{(0,7)}$.

(c) $y = 2x^2 - 3x + 1$ — The x-intercepts are $\boxed{(1,0),\ \left(\frac{1}{2},0\right)}$, and the y-intercepts are $\boxed{(0,1)}$.

Solution.

(a) For the y-intercept we set $x = 0$ and solve for y: $0 + 3y = 12$ implies $y = 4$.
For the x-intercept we set $y = 0$ and solve for x: $2x + 0 = 12$ implies $x = 6$.

(b) For the y-intercept we set $x = 0$ and solve for y: $y = 7$ implies $y = 7$.
For the x-intercept we set $y = 0$ and solve for x: $0 = 7$ has no solution.
Important note. $y = 7$ is a horizontal line. There was no need for any computation.

(c) For the y-intercept we set $x = 0$ and solve for y: $y = 2 \cdot 0^2 - 3 \cdot 0 + 1$ implies $y = 1$.
For the x-intercept we set $y = 0$ and solve for x: $0 = 2x^2 - 3x + 1$ implies $x = \frac{3 \pm \sqrt{9-8}}{4} = \frac{1}{2}, 1$. □

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