The next online test (Chapter 3/4) is 10/28 to 11/1/2017.

1 Basic laws for exponentials/logs

- $a^x \cdot a^y = a^{x+y}$
- $\frac{1}{a^x} = a^{-x}$
- $a^x \cdot b^x = (ab)^x$
- $(a^x)^y = a^{xy}$
- $\log_a(a^x) = x$ and $a^{\log_a(r)} = r$
- $\log_a(rs) = \log_a(r) + \log_a(s)$
- $\log_a\left(\frac{1}{r}\right) = -\log_a(r)$

Armin Straub
straub@southalabama.edu

2 e^x and $\ln(x)$

$2^0 = 1$, $3^0 = 1$

Observations from plots in GeoGebra:

Slope of 2^x at $x = 0$ is about 0.693.

Slope of 3^x at $x = 0$ is about 1.099.

$e \approx 2.718$ is the "magical" quantity in between.

Slope of e^x at $x = 0$ is exactly 1.

The letter e is in honor of Leonhard Euler.

$\boxed{\frac{\mathrm{d}}{\mathrm{d}x} e^x = e^x}$

$\boxed{\ln \text{ is short for } \log_e}$

"**n**atural" **l**ogarithm

$\boxed{e^{\ln(x)} = x}$

$\boxed{\ln(e^x) = x}$

Example 1. Solve $e^x = 11$.

Solution. Apply $\ln$ to both sides to get $x = \ln(11)$.

[Because $\ln(e^x) = x$.]

Example 2. Simplify $e^{x\ln(3)}$.

Solution.

$e^{x\ln(3)} = (e^{\ln(3)})^x = 3^x$

[Using $(a^r)^s = a^{rs}$ backwards.]

Example 3. Simplify $e^{3\ln(x)}$.

Solution. Same as before with 3 and x swapped:

$e^{3\ln(x)} = (e^{\ln(x)})^3 = x^3$

Armin Straub
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3 Derivatives of exponentials and logs

(chain rule)

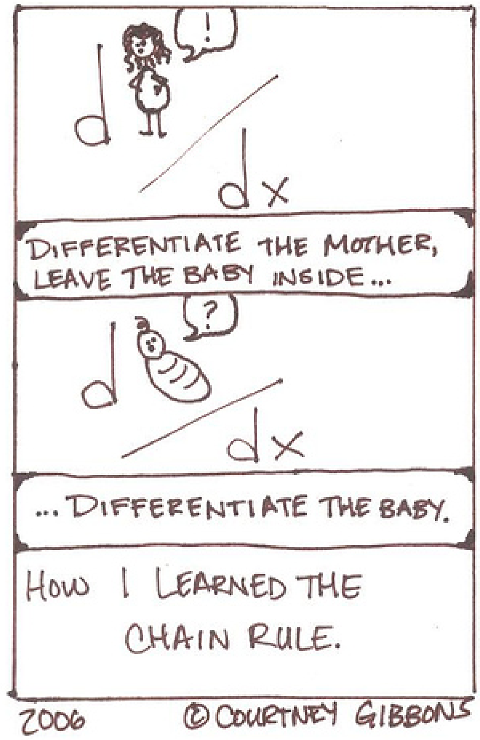
$$\frac{\mathrm{d}}{\mathrm{d}x}[f(g(x))] = f'(g(x)) \cdot g'(x)$$

(product rule)

$$\frac{\mathrm{d}}{\mathrm{d}x}[f(x)g(x)] = f'(x)g(x) + f(x)\, g'(x)$$

(exponentials/logarithms)

$$\frac{\mathrm{d}}{\mathrm{d}x}e^x = e^x \qquad \frac{\mathrm{d}}{\mathrm{d}x}\ln(x) = \frac{1}{x}$$



Armin Straub
straub@southalabama.edu

Example 4. Differentiate $h(x) = e^{x/4}$.

Solution.

Write $h(x) = f(g(x))$ with $f(x) = e^x$ and $g(x) = \frac{x}{4}$.

Then, $f'(x) = e^x$ and $g'(x) = \frac{1}{4}$.

$$\begin{aligned} h'(x) &= f'(g(x)) \cdot g'(x) \qquad \text{(chain rule)} \\ &= e^{x/4} \cdot \frac{1}{4} \\ &= \frac{1}{4} e^{x/4} \end{aligned}$$

Example 5. Differentiate $h(x) = (x^3 + 2x)e^{x/4}$.

Solution.

Write $h(x) = f(x)g(x)$ with $f(x) = x^3 + 2x$ and $g(x) = e^{x/4}$.

Then, $f'(x) = 3x^2 + 2$ and $g'(x) = \frac{1}{4} e^{x/4}$.

$$\begin{aligned} h'(x) &= f'(x)g(x) + f(x)\,g'(x) \qquad \text{(product rule)} \\ &= (3x^2 + 2)e^{x/4} + (x^3 + 2x)\left(\tfrac{1}{4} e^{x/4}\right) \\ &= \left(\tfrac{1}{4}x^3 + 3x^2 + \tfrac{1}{2}x + 2\right)e^{x/4} \end{aligned}$$

$$\boxed{\frac{\mathrm{d}}{\mathrm{d}x} e^x = e^x \qquad \frac{\mathrm{d}}{\mathrm{d}x} \ln(x) = \frac{1}{x}}$$

Example 6. Differentiate $h(x) = \ln(5x^2 + 7x)$.

Solution.

Write $h(x) = f(g(x))$ with $f(x) = \ln(x)$ and $g(x) = 5x^2 + 7x$.

Then, $f'(x) = \frac{1}{x}$ and $g'(x) = 10x + 7$.

$$\begin{aligned} h'(x) &= f'(g(x)) \cdot g'(x) \qquad \text{(chain rule)} \\ &= \frac{1}{5x^2 + 7x} \cdot (10x + 7) \\ &= \frac{10x + 7}{5x^2 + 7x} \end{aligned}$$

Assignments.

- check out Sections 4.1-6 in the book
- do "4.1, 4.2. Exponential functions and e^x" (10 questions)
- do "4.3 On the derivative of e^x." (6 questions)
- do "4.4, 4.5, 4.6. About ln(x)" (9 questions)
- take "chapter 4 quiz" (7 questions)

  All questions taken from the homework.

  5/7 questions ask for a derivative.

4 Online test after next class

- The next online test (Chapter 3/4) is 10/28 to 11/1/2017.
- Next class is used for exam prep. No new homework will be added.