1 Two tests next week

- In-class exam next week!
  - pen and paper; no calculator; no notes
  - practice problems are already posted
- Online MLP test on same topics: take 9/30–10/4
  - password will be emailed 9/29

Missed an assignment/quiz? Hurricane worries, technical issues, sleepiness, ...?

Until the in-class exam, you are able to submit all of these late.

Please aim for 100% on the homework!

- Not satisfied with a quiz so far?

  If you have no unexcused absences, send me an email before the exam with your name, section number and the (one!) quiz you would like to take again.

  [*Reminder:* Attendance is mandatory. Every absence needs to be excused!]

- Let me know about any typos!

  Any typo caught before it is fixed, will be worth a bonus (an extra attempt on a quiz of your choice).

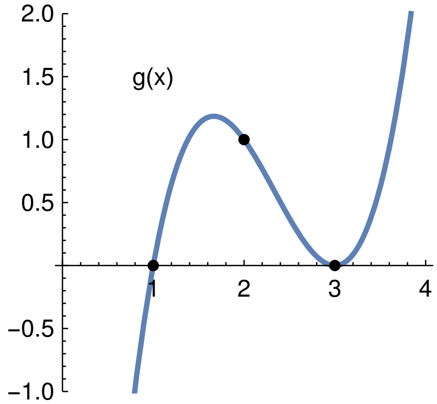
2 What to expect on the in-class exam

like practice exam but shorter

3 What to expect on the online test

take "Test on chapters 1 and 2" (10 questions, 60min)

- power rule
- determine slope of curve at x (using power rule)
- differentiate polynomial
- estimate, say, $C(43.5)$ given $C(42)$ and $C'(42)$
- identify graphs with positive/negative first derivative
- identify graphs with positive/negative second derivative
- determine sign of f, f', f'' at certain points
- optimization problem (as in 2.5 homework)
- given cost function, minimize marginal cost
- economic optimization problem (as in 2.7 homework)



Example 1. Let $g(x)$ be the function in the graph.

(a) If $g(x) = f'(x)$, what can you say about $f(x)$ at $x = 1$? At $x = 2$? At $x = 3$?

(b) If $g(x) = f''(x)$, what can you say about $f(x)$ at $x = 1$? At $x = 2$? At $x = 3$?

Solution.

(a) At $x = 1$: $f'(1) = 0$ and f' changes from $-$ to $+$

$\Longrightarrow$ $f(x)$ has local min at $x = 1$

At $x = 2$: $f'(2) > 0$ $\Longrightarrow$ f is increasing at $x = 2$

At $x = 3$: $f'(3) = 0$ but f' does not change sign

$\Longrightarrow$ $f(x)$ has no local min/max at $x = 3$

In fact, we can also read off the concavity of f (note that $f'' = g'$):

At $x = 1$: $f''(1) > 0$ $\Longrightarrow$ f is concave up at $x = 1$

At $x = 2$: $f''(2) < 0$ $\Longrightarrow$ f is concave down at $x = 2$

At $x = 3$: $f''(3) = 0$ and f'' changes from $-$ to $+$

$\Longrightarrow$ $f(x)$ has an inflection point at $x = 3$

(b) If $g(x) = f''(x)$, what can you say about $f(x)$ at $x = 1$? At $x = 2$? At $x = 3$?

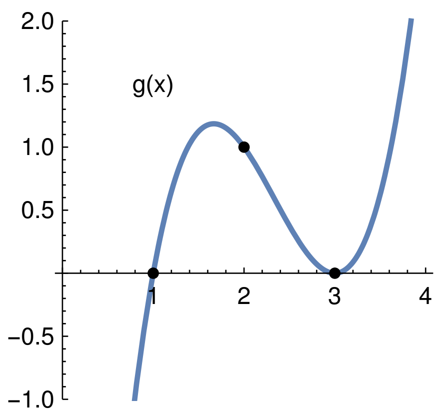
At $x = 1$: $f''(1) = 0$ and f'' changes from $-$ to $+$

$\Longrightarrow$ $f(x)$ has an inflection point at $x = 1$

At $x = 2$: $f''(2) > 0$ $\Longrightarrow$ f is concave up at $x = 2$

At $x = 3$: $f''(3) = 0$ but f'' does not change sign

$\Longrightarrow$ $f(x)$ has no inflection point at $x = 3$



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Quick warm-up for the in-class (and online) exam:

Example 2. Given the cost function $C(x) = x^3 - 6x^2 + 13x + 18$, find the minimal marginal cost.

Solution.

The marginal cost function is $M(x) = C'(x) = 3x^2 - 12x + 13$.

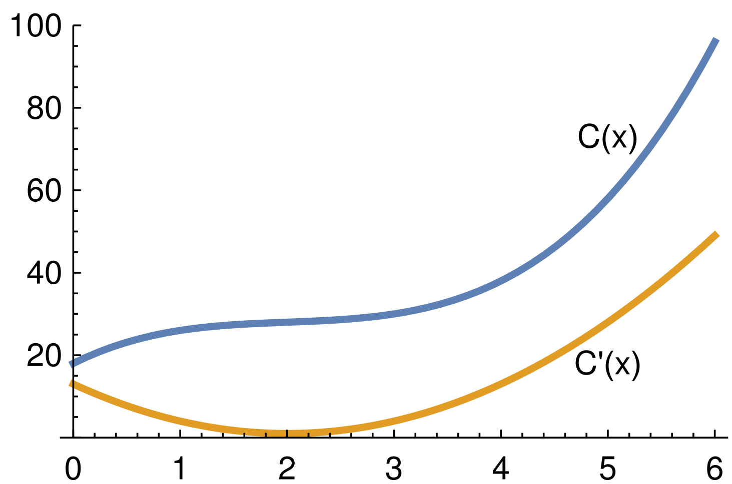
We need to find the minimum of $M(x)$.

$M'(x) = 6x - 12$

Solving $M'(x) = 0$, we find $x = 2$.

Assuming the problem is well-posed, $x = 2$ must be the minimum because it is the only candidate.

The minimal marginal cost is $M(2) = 1$.



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A problem like this will surely show up in the online test.

Example 3. At a price of 6 dollars, 50 beers were sold per night at a local cinema. When the price was raised to 7 dollars, sales dropped to 40 beers.

(a) Assuming a linear demand curve, which price maximizes revenue?

Solution.

(setup) p prize per beer, x number of beer sold

Revenue is $R(x) = p \cdot x$. **(objective equation)**

Linear demand means that $p = ax + b$ (a line!) for some a, b.

We know that $(x_1, p_1) = (50, 6)$ and $(x_2, p_2) = (40, 7)$.

Hence, $p - 6 = \underbrace{\text{slope}}_{\frac{p_2 - p_1}{x_2 - x_1} = \frac{1}{-10}} (x - 50)$.

This simplifies to $p = 11 - \frac{1}{10}x$. **(constraint equation)**

Revenue is $R(x) = p \cdot x = \left(11 - \frac{1}{10}x\right) \cdot x = 11x - \frac{1}{10}x^2$.

(find max of $R(x)$) $R'(x) = 11 - \frac{2}{10}x$

Solving $R'(x) = 11 - \frac{1}{5}x = 0$, we find $x = 55$.

The corresponding price is $p = 11 - \frac{1}{10} \cdot 55 = 5.5$ dollars.

(Then the revenue is $R(55) = 5.5 \cdot 55 = 302.5$ dollars.)

(b) Suppose the cinema has fixed costs of 100 dollars per night for selling beer, and variable costs of 2 dollars per beer. Find the price that maximizes profit.

Solution.

(setup) p prize per beer, x number of beer sold

Cost is $C(x) = 100 + 2x$.

As before, revenue is $R(x) = 11x - \frac{1}{10}x^2$.

Profit is $P(x) = R(x) - C(x) = 9x - \frac{1}{10}x^2 - 100$.

(find max of $P(x)$) $P'(x) = 9 - \frac{2}{10}x$

Solving $P'(x) = 9 - \frac{1}{5}x = 0$, we find $x = 45$.

The corresponding price is $p = 11 - \frac{1}{10} \cdot 45 = 6.5$ dollars.

(Then the profit is $P(45) = 102.5$ dollars.)

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