Quiz #4

MATH 125 — Calculus 1
Tuesday, Feb 12

Please print your name:

Problem 1. (6+2 points) [Show work!]

(a) Using the limit definition, compute $f'(x)$ for $f(x) = \frac{1}{x}$.

(b) Determine the line tangent to the graph of $f(x)$ at $x = 2$.

Solution.

(a) We need to determine $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ for $f(x) = \dfrac{1}{x}$.

Note that

$$f(x+h) - f(x) = \frac{1}{x+h} - \frac{1}{x} = \frac{x - (x+h)}{(x+h)x} = \frac{-h}{(x+h)x},$$

so that

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{-h}{(x+h)xh} = \lim_{h \to 0} \frac{-1}{(x+h)x} = \frac{-1}{(x+0)x} = -\frac{1}{x^2}.$$

(b) From the first part, the slope of that line is $f'(2) = -\frac{1}{4}$. It also passes through $(2, f(2)) = \left(2, \frac{1}{2}\right)$.

Hence, it has the equation $y - \frac{1}{2} = -\frac{1}{4}(x-2)$, which simplifies to $y = -\frac{1}{4}x + 1$. □

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