Prime factorizations

DEF An integer $p > 1$ is a **prime** if its only positive divisors are 1 and p.

- Let p be a prime. Then: and only then

$$p \mid ab \implies p \mid a \text{ or } p \mid b$$

EG $\underset{2\cdot 3}{6} \mid 8\cdot 21$ but $6 \nmid 8$ and $6 \nmid 21$

But: $3 \mid 8 \cdot 21 \implies 3 \mid 21$

- **Fundamental theorem of arithmetic:**
Every integer $n > 1$ can be written as a product of primes. This factorization is unique (apart from ordering)

EG $2020 = 2^2 \cdot 5 \cdot 101$ prime factorization

not divisible by $2, 3, 5, 7$ $\implies$ 101 prime $\quad 11 \cdot 11 > 101$

Advanced comment:

integers $\rightsquigarrow$ generalized integers

$a \in \mathbb{Z}$ $\qquad a + b\, i\sqrt{5} \quad a, b \in \mathbb{Z}$

then: factorization is not unique

$$6 = 2 \cdot 3$$
$$= \underbrace{(1 + i\sqrt{5})(1 - i\sqrt{5})}_{1^2 - (i\sqrt{5})^2 = 1 + 5}$$