

Midterm #1

Please print your name:

No notes or tools of any kind are permitted.

There are 33 points in total.

You need to show work to receive full credit.

Good luck!

Problem 1. (4+2+2 points)

- (a) Let $d = \gcd(16, 23)$. Using the Euclidean algorithm, find integers x, y such that $16x + 23y = d$.
- (b) Find the modular inverse of 16 modulo 23.
- (c) Solve $16x \equiv 3 \pmod{23}$.

Problem 2. (10 points)

(a) The remainder of 314159 modulo 11 is

(b) $2^{-1} \pmod{19}$ is

(c) Complete the following to a complete set of residues modulo 5: 11, -10, 9, 2,

(d) The number 26 in base 2 is

(e) The number 26 in base 3 is

(f) List all invertible residues modulo 12:

(g) The residue x is invertible modulo n if and only if

(h) For which values of k has the diophantine equation $15x + 9y = k$ at least one integer solution?

(i) How many solutions does $7x \equiv 12 \pmod{60}$ have modulo 60?

How many solutions does $6x \equiv 12 \pmod{60}$ have modulo 60?

How many solutions does $6x \equiv 2 \pmod{60}$ have modulo 60?

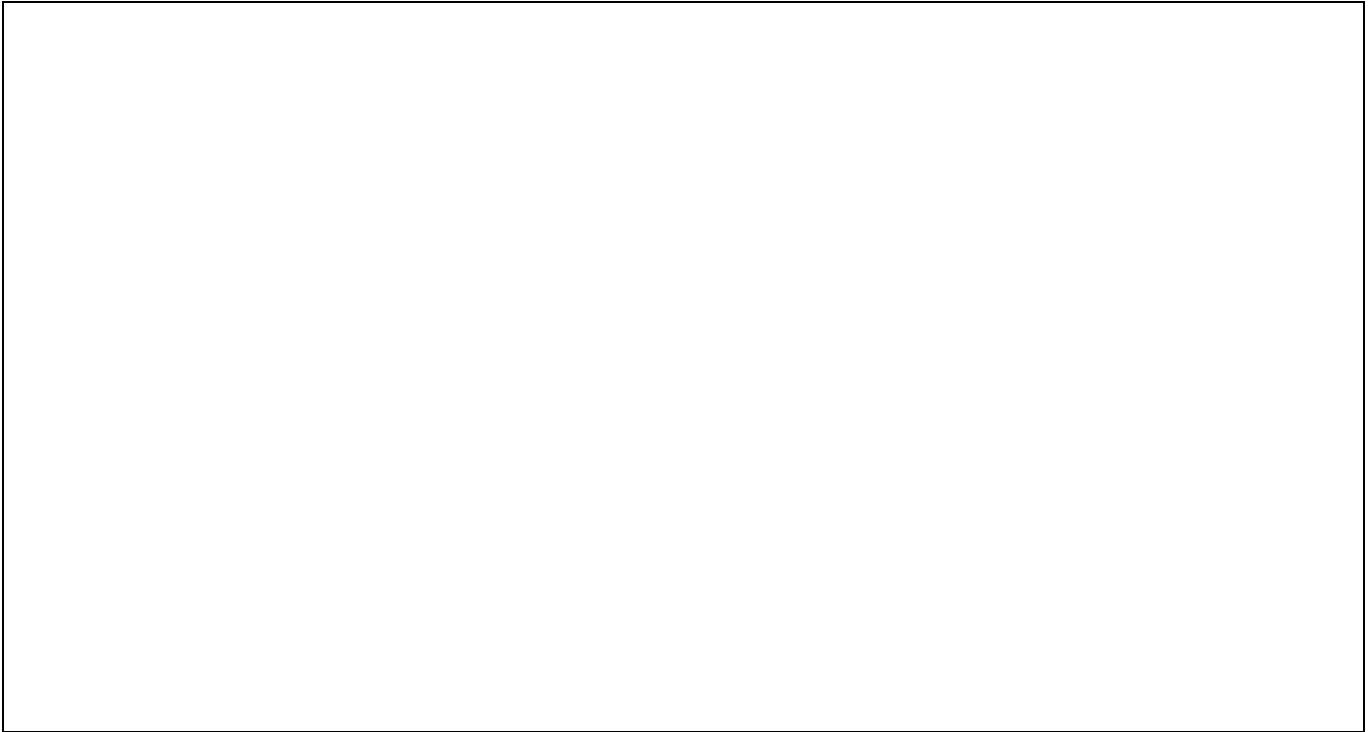
(j) Up to x , there are roughly

many primes.

(scratch space)

Problem 3. (5 points) Determine $25^{3630} \pmod{19}$.

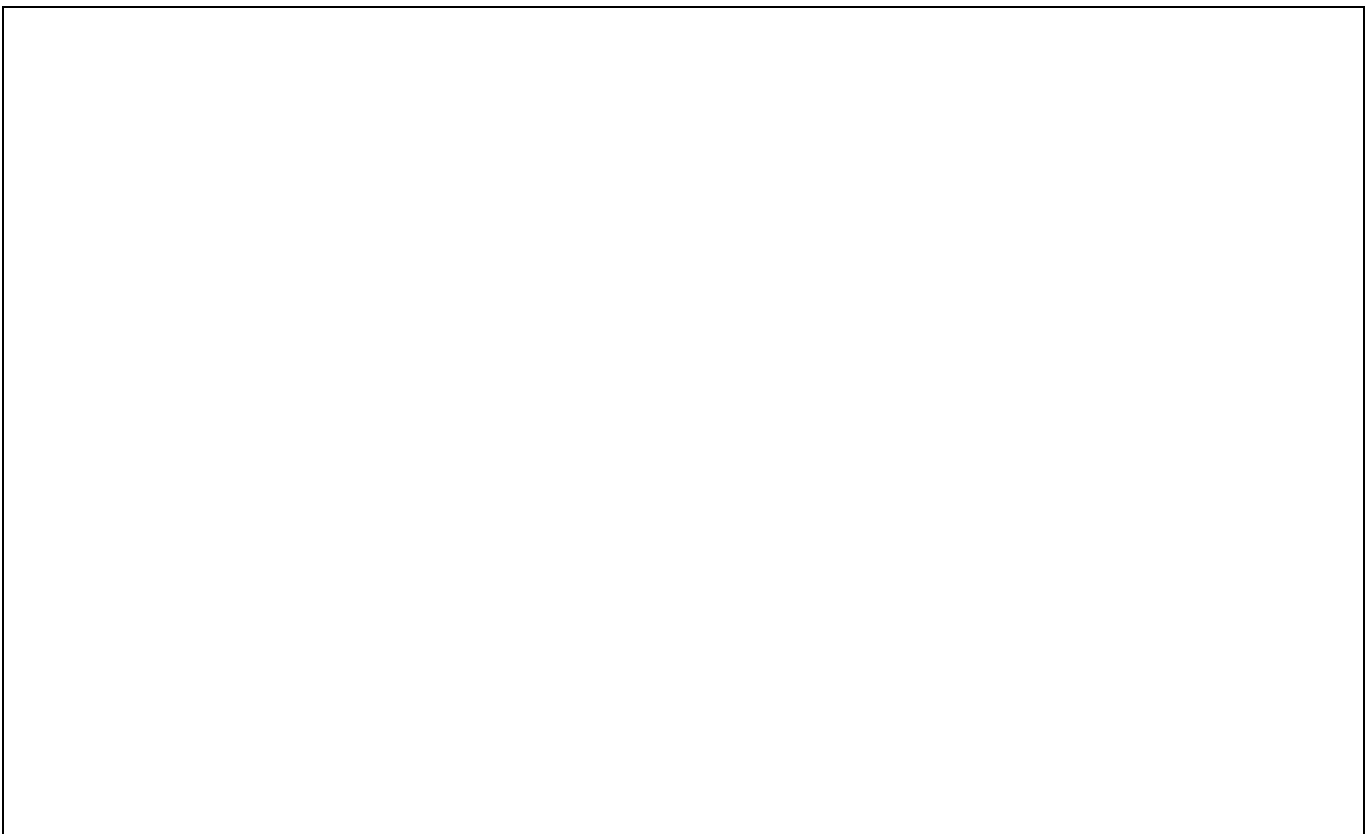
Carefully show all steps!



Problem 4. (4 points) Solve the following system of congruences:

$$2x + y \equiv 3 \pmod{15}$$

$$x - 3y \equiv 1 \pmod{15}$$



Problem 5. (6 points) Using the Chinese remainder theorem, determine all solutions to $x^2 \equiv 1 \pmod{55}$.

(extra scratch paper)