Midterm #2

MATH 311 — Intro to Number Theory
Tuesday, Nov 13

Please print your name:

No notes or tools of any kind are permitted. There are 28 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. **(4 points)** Using the Chinese remainder theorem, determine all solutions to $x^2 \equiv 1 \pmod{77}$.

Problem 2. **(2 points)** Suppose that $x^a \equiv 1 \pmod{n}$ and $x^b \equiv 1 \pmod{n}$. Show that $x^{\gcd(a,b)} \equiv 1 \pmod{n}$.

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Problem 3. (3 points) What is the last (decimal) digit of 7^{123456}?

Problem 4. (4 points) Obviously, 15 is not a prime. Is 7 a Fermat liar modulo 15? Is 4 a Fermat liar modulo 15?

Problem 5. (3 points) Briefly outline the Fermat primality test.

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Problem 6. (12 points)

(a) Among the numbers $1, 2, \ldots, 54$, how many are coprime to 54?

(b) If $n = p^2 q$, for distinct primes p, q, then $\phi(n) =$.

(c) How many solutions does the congruence $x^2 \equiv 4 \pmod{105}$ have?

(d) How many solutions does the congruence $x^2 \equiv 4 \pmod{210}$ have?

(e) How many solutions does the congruence $x^2 \equiv 4 \pmod{3135}$ have? $(3135 = 3 \cdot 5 \cdot 11 \cdot 19)$

(f) $x = 30$ is a solution to $\begin{array}{l} x \equiv 30 \pmod{100} \\ x \equiv 8 \pmod{11} \end{array}$. Another positive solution is .

(g) The multiplicative order of $3 \pmod{11}$ is .

(h) The multiplicative order of $x \pmod{88}$ divides .

(i) The primitive roots modulo 7 are .

(j) If $x \pmod{n}$ has multiplicative order k, then x^{2018} has multiplicative order .

(k) What is the number of invertible residues modulo 29?

(l) What is the number of primitive roots modulo the prime 89?

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(extra scratch paper)