

Midterm #1

Please print your name:

No notes or tools of any kind are permitted.

There are 28 points in total.

You need to show work to receive full credit.

Good luck!

Problem 1. (4+2+2 points)

- (a) Let $d = \gcd(17, 23)$. Using the Euclidean algorithm, find integers x, y such that $17x + 23y = d$.
- (b) Find the modular inverse of 17 modulo 23.
- (c) Solve $17x \equiv 10 \pmod{23}$.

Problem 2. (9 points)

(a) The remainder of 124816 modulo 11 is

(b) $5^{-1} \pmod{7}$ is

(c) Complete the following to a complete set of residues modulo 6: 3, -1, 6, 8, 4,

(d) The number 55 in base 5 is

(e) List all invertible residues modulo 12:

(f) The residue x is invertible modulo n if and only if

(g) For which values of k has the diophantine equation $21x + 6y = k$ at least one integer solution?

(h) How many solutions does $3x \equiv 2 \pmod{50}$ have modulo 50?

How many solutions does $5x \equiv 2 \pmod{50}$ have modulo 50?

How many solutions does $5x \equiv 20 \pmod{50}$ have modulo 50?

(scratch space)

Problem 3. (2 points) Carefully state Fermat's little theorem.

Problem 4. (5 points) Determine $40^{1612} \pmod{17}$.

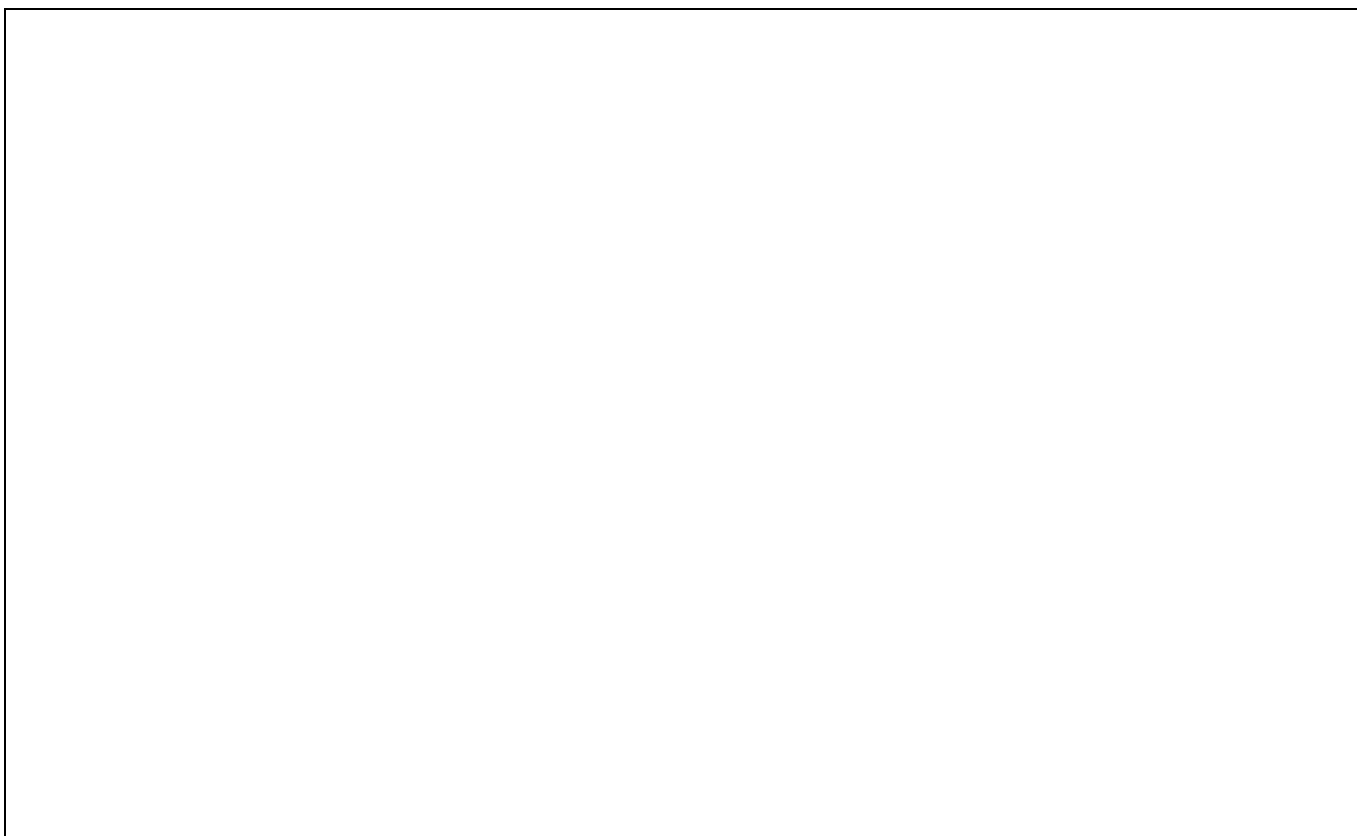
Carefully show all steps!



Problem 5. (4 points) Solve the following system of congruences:

$$2x + y \equiv 3 \pmod{15}$$

$$x - 3y \equiv 1 \pmod{15}$$



(extra scratch paper)