

Midterm #3

Please print your name:

No notes or tools of any kind are permitted.

There are 26 points in total.

You need to show work to receive full credit.

Good luck!

Problem 1. (3 points) Obviously, 15 is not a prime. Is 15 a pseudoprime to the base 7?

Problem 2. (3 points) Briefly outline the Fermat primality test.

Problem 3. (3 points) What is the last (decimal) digit of 3^{14159} ?

Problem 4. (2 points) Carefully state Wilson's theorem.

Problem 5. (3 points) Express the number $\frac{89}{69}$ as a simple continued fraction.

Problem 6. (2+2 points)

- (a) Determine the convergents C_0, C_1, C_2, C_3 of the infinite continued fraction $[1; 4, 1, 4, 1, 4, 1, 4, \dots]$.
- (b) Which number is represented by the infinite continued fraction $[1; 4, 1, 4, 1, 4, 1, 4, \dots]$?

Problem 7. (1 point) Among the numbers $1, 2, \dots, 54$, how many are coprime to 54?

Problem 8. (1 point) List all (nonzero) quadratic residues modulo 7.

Problem 9. (2 points) Suppose that $x^a \equiv 1 \pmod{n}$ and $x^b \equiv 1 \pmod{n}$. Show that $x^{\gcd(a,b)} \equiv 1 \pmod{n}$.

Problem 10. (4 points) Using the Chinese remainder theorem, determine all solutions to $x^2 \equiv 1 \pmod{55}$.

(extra scratch paper)