

# Midterm #2

Please print your name:

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No notes or tools of any kind are permitted.

There are 25 points in total.

You need to show work to receive full credit.

**Good luck!**

**Problem 1. (warmup, 4 points)**

(a) The remainder of 10202017 modulo 11 is

(b) Complete the following to a complete set of residues modulo 7:

(c) The number 51 in base 7 is

(d) List all primitive roots modulo 5:

(scratch space)

**Problem 2. (warmup, 2 points)** Carefully state Fermat's little theorem.

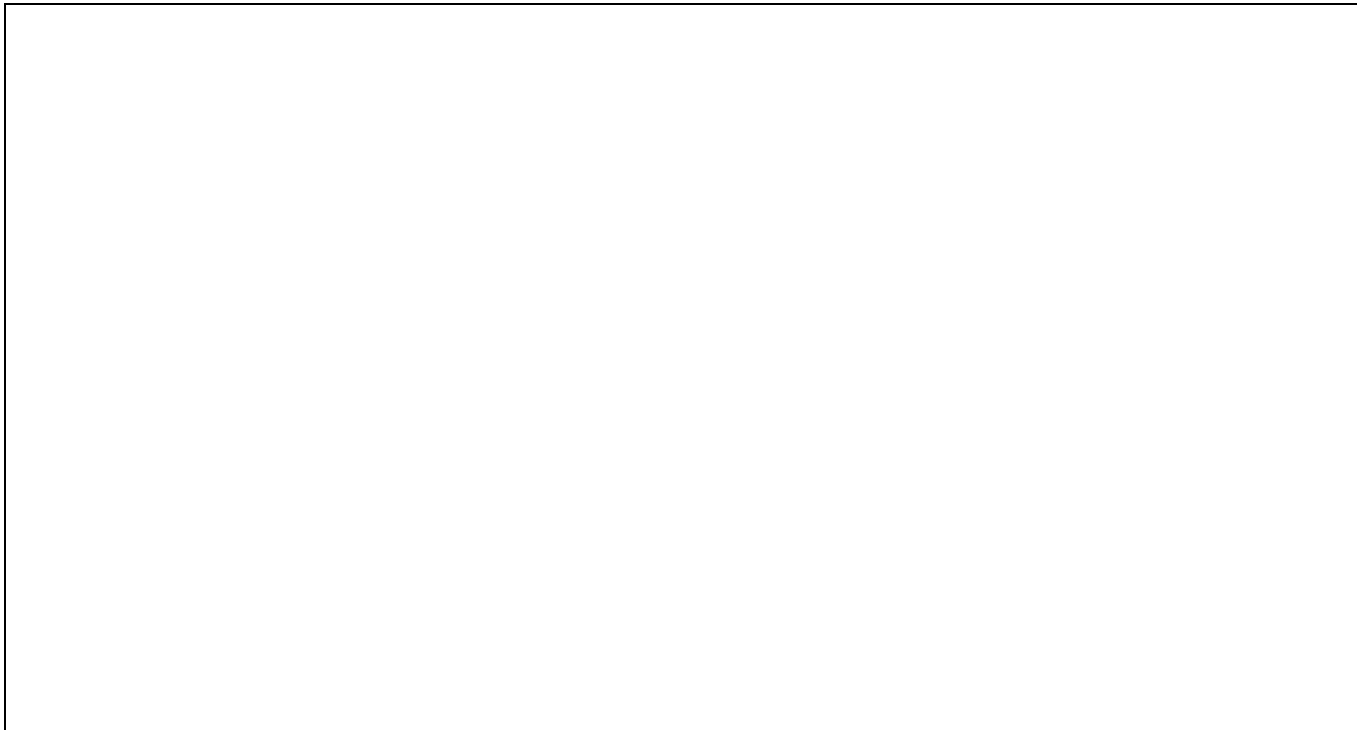
**Problem 3. (3 points)** Determine whether  $31^{41} + 59^{26} + 53^5$  is divisible by 5.

Carefully show your steps!

**Problem 4. (3+1 points)**

(a) Find the modular inverse of 10 modulo 43.

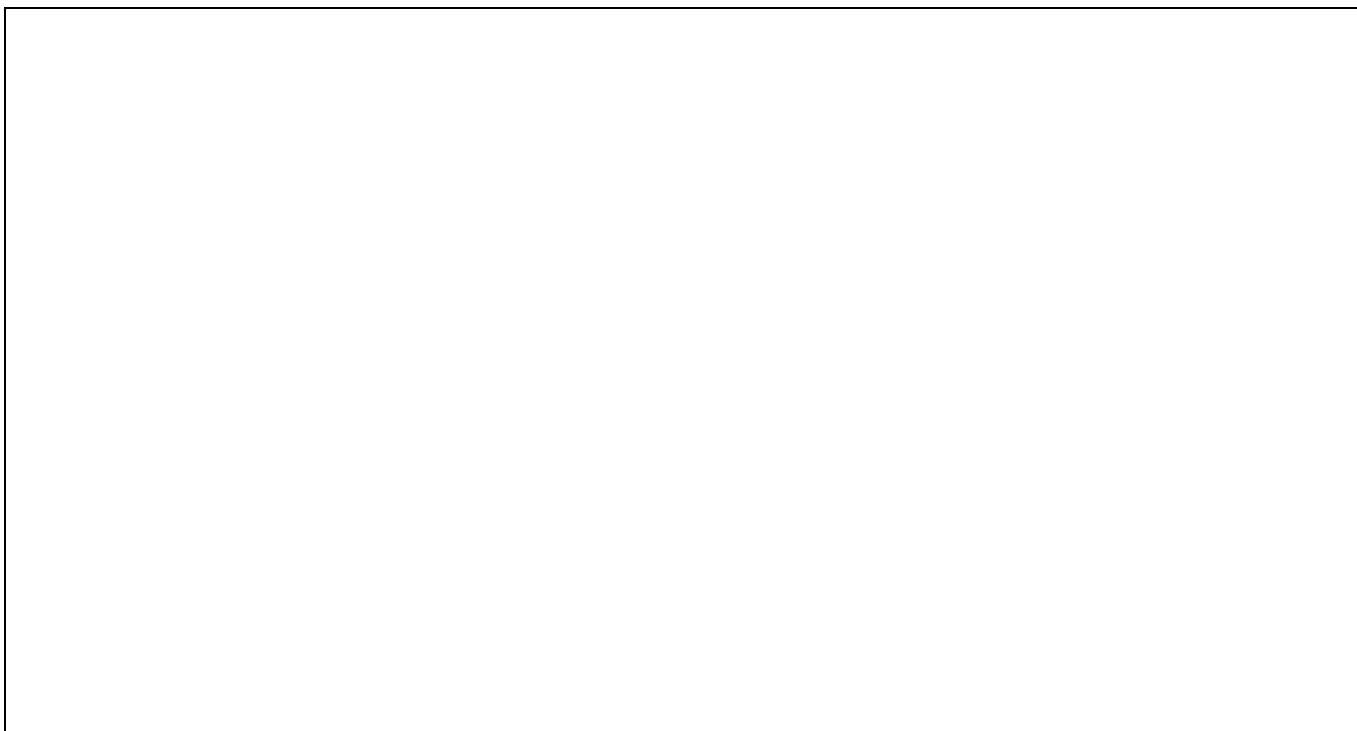
(b) Solve  $10x \equiv 4 \pmod{43}$ .



**Problem 5. (4 points)** Solve the following system of congruences:

$$3x - y \equiv 1 \pmod{15}$$

$$x + 2y \equiv 4 \pmod{15}$$



**Problem 6. (3 points)** Using binary exponentiation, compute  $6^{13} \pmod{17}$ .

Carefully show all steps!

**Problem 7. (4+1 points)**

- (a) Find the smallest positive integer  $x$  simultaneously solving the three congruences
- $$\begin{aligned} x &\equiv 1 \pmod{3}, \\ x &\equiv 4 \pmod{7}, \\ x &\equiv 1 \pmod{10}. \end{aligned}$$
- (b) The next largest solution  $x$  to the above congruences is .

(extra scratch paper)