Midterm #1

MATH 311 — Intro to Number Theory
Tuesday, Sep 20

Please print your name:

No notes or tools of any kind are permitted. There are 24 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (warmup, 2 points) Write down the first 5 rows of the Pascal triangle. Use this to expand $(x+y)^5$.

Problem 2. (warmup, 2 points) The sequence $A(n)$ is defined by $A(1)=1$, $A(2)=1$ and

$$A(n)=n\cdot A(n-1)-A(n-2)$$

for $n>2$. Determine $A(4)$.

(This sequence appears, for instance, in the context of Bessel functions [D.H. Lehmer, *Annals of Mathematics* 33(1), 1932].)

Problem 3. (1+3 points)

(a) Gauss' summation formula states that $\sum_{k=1}^{n} k = \boxed{}$.

(b) Derive a formula for the sum $\sum_{k=0}^{n}(3k+2)$ from Gauss' summation formula.

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Problem 4. (1 point) Which positive integers n have the following property: for all integers a, b,

$$\text{if } n|ab, \quad \text{then } n|a \text{ or } n|b.$$

Problem 5. (3 points) Which are the possible remainders that the square of an integer leaves upon division by 4?

Problem 6. (4+1 points)

(a) Find $d = \gcd(32, 50)$. Using the Euclidean algorithm, find integers x, y such that $32x + 50y = d$.

(b) For which values of k has the diophantine equation $32x + 50y = k$ at least one integer solution?

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Problem 7. (3 points) Determine all solutions of $8x + 14y = 4$ with x and y integers.

Problem 8. (4 points) Using induction, prove that $15|2^{4n} - 1$ for all integers $n \geqslant 0$.

Carefully state all steps of inductions to receive full credit!

(extra scratch paper)