Homework #2

MATH 311 — Intro to Number Theory
due in class on Thursday, Sep 8

Please print your name:

These problems are not suited to be done last minute!
Also, if you start early, you can consult with me if you should get stuck.

Problem 1.

(a) Write down the first 6 rows of the Pascal triangle.

(b) Expand $(x+y)^6$.

(c) For each row in Pascal's triangle, compute the sum of all entries in that row. Conjecture a formula.

(d) Prove that formula using the binomial theorem.

Problem 2. In class, we gave a combinatorial argument showing that

$$\binom{n+1}{3} = \binom{n}{2} + \binom{n-1}{2} + \ldots + \binom{3}{2} + \binom{2}{2}.$$

Prove that formula using induction.

Problem 3. Which are the possible remainders that the square of an integer leaves upon division by 5?

Problem 4.

(a) Prove or disprove: for any integer x, one of the integers x, $x+2$, $x+4$ is divisible by 3.

(b) Prove or disprove: for any integer x, one of the integers x, $x+2$, $x+8$ is divisible by 3.

(c) Prove or disprove: for any integer x, one of the integers x, $x+5$, $x+7$ is divisible by 3.

(d) Formulate a (necessary and sufficient) condition on a, b such that the following statement is true: for any integer x, one of the integers x, $x+a$, $x+b$ is divisible by 3.

Problem 5. Let $n \geqslant 0$ be an integer. Using induction, prove the following divisibility statements:

(a) $8 | 5^{2n} + 7$ *Hint:* $5^{2(n+1)} + 7 = 5^2(5^{2n} + 7) + (7 - 5^2 \cdot 7)$

(b) $15 | 2^{4n} - 1$

Armin Straub
straub@southalabama.edu