## Midterm #1

Please print your name:

No notes, calculators or tools of any kind are permitted. There are 29 points in total. You need to show work to receive full credit.

## Good luck!

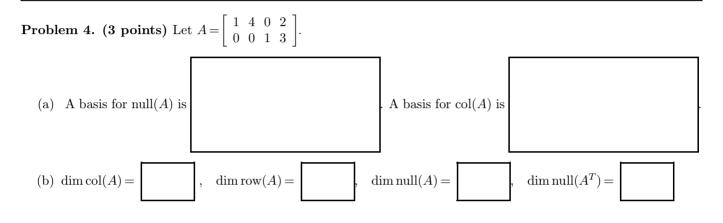
## Problem 1. (6 points)

- (a) Find the least squares solution to  $\begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} 2 \\ 4 \\ -1 \\ 2 \end{bmatrix}.$
- (b) Determine the least squares line for the data points (-1, 2), (0, 4), (1, -1), (1, 2).

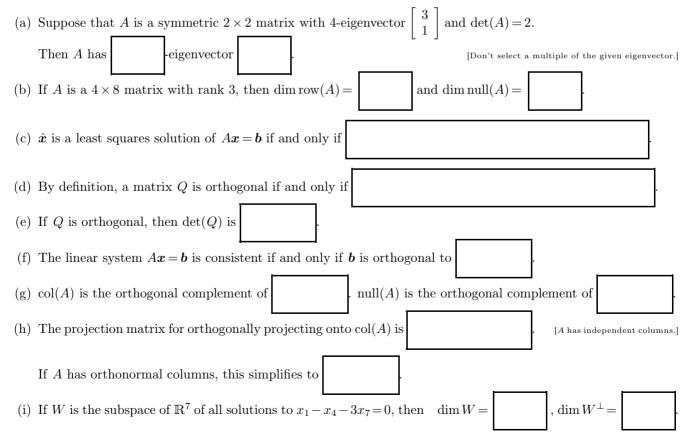
## Problem 2. (8 points)

- (a) Using Gram–Schmidt, obtain an orthonormal basis for  $W = \operatorname{span} \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} \right\}$ . (b) Determine the orthogonal projection of  $\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$  onto W. (c) Determine the QR decomposition of the matrix  $A = \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$ .

**Problem 3. (3 points)** We want to find values for the parameters a, b, c such that  $z = ax^2 + b + \frac{c}{y}$  best fits some given points  $(x_1, y_1, z_1), (x_2, y_2, z_2), \ldots$  Set up a linear system such that  $[a, b, c]^T$  is a least squares solution.



Problem 5. (9 points) Fill in the blanks.



(extra scratch paper)