Midterm #1

MATH 316 — Linear Algebra II
Friday, Feb 17, 2023

Please print your name:

No notes, calculators or tools of any kind are permitted. There are 29 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (6 points)

(a) Find the least squares solution to $\begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} 2 \\ 4 \\ -1 \\ 2 \end{bmatrix}$.

(b) Determine the least squares line for the data points $(-1, 2), (0, 4), (1, -1), (1, 2)$.

Armin Straub
straub@southalabama.edu

Problem 2. (8 points)

(a) Using Gram–Schmidt, obtain an orthonormal basis for $W = \operatorname{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} \right\}$.

(b) Determine the orthogonal projection of $\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$ onto W.

(c) Determine the QR decomposition of the matrix $A = \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$.

Problem 3. (3 points) We want to find values for the parameters a, b, c such that $z = ax^2 + b + \frac{c}{y}$ best fits some given points $(x_1, y_1, z_1), (x_2, y_2, z_2), \ldots$ Set up a linear system such that $[a, b, c]^T$ is a least squares solution.

Problem 4. (3 points) Let $A = \begin{bmatrix} 1 & 4 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$.

(a) A basis for null(A) is ______. A basis for col(A) is ______.

(b) $\dim \operatorname{col}(A) =$ ______, $\dim \operatorname{row}(A) =$ ______, $\dim \operatorname{null}(A) =$ ______, $\dim \operatorname{null}(A^T) =$ ______

Problem 5. (9 points) Fill in the blanks.

(a) Suppose that A is a symmetric 2×2 matrix with 4-eigenvector $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $\det(A) = 2$.

Then A has ______-eigenvector ______. [Don't select a multiple of the given eigenvector.]

(b) If A is a 4×8 matrix with rank 3, then $\dim \operatorname{row}(A) =$ ______ and $\dim \operatorname{null}(A) =$ ______.

(c) $\hat{\boldsymbol{x}}$ is a least squares solution of $A\boldsymbol{x} = \boldsymbol{b}$ if and only if ______.

(d) By definition, a matrix Q is orthogonal if and only if ______.

(e) If Q is orthogonal, then $\det(Q)$ is ______.

(f) The linear system $A\boldsymbol{x} = \boldsymbol{b}$ is consistent if and only if $\boldsymbol{b}$ is orthogonal to ______.

(g) col(A) is the orthogonal complement of ______. null(A) is the orthogonal complement of ______.

(h) The projection matrix for orthogonally projecting onto col(A) is ______. [A has independent columns.]

If A has orthonormal columns, this simplifies to ______.

(i) If W is the subspace of $\mathbb{R}^7$ of all solutions to $x_1 - x_4 - 3x_7 = 0$, then $\dim W =$ ______, $\dim W^{\perp} =$ ______.

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(extra scratch paper)