Midterm #2 — MON, 4/15

exam: 8^{00} – 8^{55} AM — upload work by 9^{30} AM (PDF)

format
- like for first exam
- show-your-work problems ~ 2
- short answer problems ~ 8-10
  no "real" work needed

practice
- review HW
- practice problems + solutions

tools
- calculators allowed — but: show work
- notes allowed — but: watch time

Questions?

previous main theme: ORTHOGONALITY

current main theme: DIAGONALIZATION $\quad A = PDP^{-1}$

- powers of matrix $\quad A^n = PD^nP^{-1}$
- application: Markov chains
- application: PageRank
- application: recurrences
  - Binet-like formulas
  - asymptotic growth $\left(\lim_{n\to\infty} \frac{a_{n+1}}{a_n}\right)$
- application: systems of DEs $\quad e^{At} = Pe^{Dt}P^{-1}$
  - matrix exponential
  - convert higher-order DEs to systems
- non-diagonalizable $\rightsquigarrow$ Jordan normal form
  - EG $\quad e^{\begin{pmatrix} 7 & 1 \\ & 7 \end{pmatrix} t}$
  - # of possible JNFs
- projections, reflections, rotations
- complex numbers
  - transpose A^T $\rightsquigarrow$ conjugate transpose A^*

EG $\quad y' = \begin{pmatrix} 7 & 6 \\ -9 & -8 \end{pmatrix} y \qquad y(0) = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$

$y' = Ay$

$y_1' = 11y_1 + 8y_2$
$y_2' = -12y_1 - 9y_2$

$$\Longrightarrow \quad y = \underbrace{e^{At}}_{2\times 2} \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$= Pe^{Dt}P^{-1} \begin{bmatrix} 3 \\ 0 \end{bmatrix} \qquad A = PDP^{-1} \quad e^{At} = Pe^{Dt}P^{-1}$$

char poly : $\begin{vmatrix} 7-\lambda & 6 \\ -9 & -8-\lambda \end{vmatrix} = (7-\lambda)(-8-\lambda) - 6\cdot(-9)$

$= \lambda^2 + \lambda - 2 \longrightarrow \lambda = -2, 1$

-2-eigenspace $= \text{null} \begin{pmatrix} 9 & 6 \\ -9 & -6 \end{pmatrix}$ $\qquad D = \begin{bmatrix} -2 & \\ & 1 \end{bmatrix}$

$\rightsquigarrow \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ or: $\begin{bmatrix} -2/3 \\ 1 \end{bmatrix}$ $\qquad e^{Dt} = \begin{bmatrix} e^{-2t} & \\ & e^{t} \end{bmatrix}$

1-eigenspace $= \text{null} \begin{pmatrix} 6 & 6 \\ -9 & -9 \end{pmatrix}$

$\rightsquigarrow \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $\qquad P = \begin{bmatrix} -2 & -1 \\ 3 & 1 \end{bmatrix}$

$$\Longrightarrow \quad y = \underbrace{\begin{bmatrix} -2 & -1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} e^{-2t} & \\ & e^{t} \end{bmatrix}}_{= \begin{bmatrix} -2e^{-2t} & -e^{t} \\ 3e^{-2t} & e^{t} \end{bmatrix}} \underbrace{\frac{1}{1} \begin{bmatrix} 1 & 1 \\ -3 & -2 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \end{bmatrix}}_{= \begin{bmatrix} 3 \\ -9 \end{bmatrix}}$$

$$= \begin{bmatrix} -6e^{-2t} + 9e^{t} \\ 9e^{-2t} - 9e^{t} \end{bmatrix}$$

EG A reflection through $\text{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 3 \end{bmatrix} \right\}$. $= W$

Diagonalize: $A = PDP^{-1}$

1-eigenspace $= W$ dim $= 2$

-1-eigenspace $= W^{\perp}$ dim $= 1$

$$D = \begin{bmatrix} 1 & & \\ & 1 & \\ & & -1 \end{bmatrix} \qquad P = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 4 & -3/4 \\ -2 & 3 & 1 \end{bmatrix}$$

$$W^{\perp} = \text{null}\left(\begin{bmatrix} 1 & 0 & -2 \\ 0 & 4 & 3 \end{bmatrix} \right)$$

free

basis: $\begin{bmatrix} 2 \\ -3/4 \\ 1 \end{bmatrix}$

$4x_2 + 3x_3 = 0$