

$$\text{EG } y' = \underbrace{\begin{bmatrix} 2 & 0 & 0 \\ -1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}}_A y \quad y(0) = \underbrace{\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}}_{y_0}$$

$$y(t) = e^{At} \cdot y_0$$

matrix exponential

• e^{Dt} easy

• $A = PDP^{-1}$
 $\Rightarrow e^{At} = P e^{Dt} P^{-1}$

$$A = PDP^{-1}$$

with $D = \begin{bmatrix} 2 & & \\ & 2 & \\ & & 4 \end{bmatrix}$ $P = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

$$y(t) = P e^{Dt} P^{-1} y_0$$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} e^{2t} & & \\ & e^{2t} & \\ & & e^{4t} \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} e^{2t} \\ e^{2t} + e^{4t} \\ e^{4t} \end{bmatrix}$$

check: $y_2' = -y_1 + 3y_2 + y_3$

$$(e^{2t} + e^{4t})' \stackrel{?}{=} -e^{2t} + 3(e^{2t} + e^{4t}) + e^{4t}$$

$$2e^{2t} + 4e^{4t} \quad \checkmark \quad 2e^{2t} + 4e^{4t}$$