$$\boxed{\begin{array}{l} y' = ay \qquad y(0) = y_0 \\ \text{has unique solution} \quad y(t) = e^{at} \cdot y_0 \end{array}}$$

EG $y' = 3y$, $y(0) = 7 \implies y(t) = 7e^{3t}$

<u>next</u> $\underset{\text{number}}{a} \rightsquigarrow \underset{n \times n}{A}$

$$\begin{aligned} y_1' &= 2y_1 & y_1(0) &= 1 \\ y_2' &= -y_1 + 3y_2 + y_3 & y_2(0) &= 0 \\ y_3' &= -y_1 + y_2 + 3y_3 & y_3(0) &= 2 \end{aligned}$$

$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

in matrix form:

$$y' = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix} y \qquad y(0) = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

$$y' = Ay \qquad y(0) = y_0$$

has unique solution $y(t) = e^{At} \cdot y_0$

e^{At} matrix exponential

How to compute?

① e^{Dt} <u>easy</u>

$$e^{\left[\begin{smallmatrix} 2 & 0 \\ 0 & 5 \end{smallmatrix}\right] t} = \begin{bmatrix} e^{2t} & 0 \\ 0 & e^{5t} \end{bmatrix}$$

② $A = PDP^{-1}$

$\implies e^{At} = P e^{Dt} P^{-1}$

$$e^t = \sum_{n=0}^{\infty} \frac{t^n}{n!} = 1 + t + \frac{t^2}{2} + \frac{t^3}{6} + \cdots$$

How to compute A^n?

① D^n <u>easy</u>

$$\begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}^n = \begin{bmatrix} 2^n & 0 \\ 0 & 5^n \end{bmatrix}$$

② $A = PDP^{-1}$

$\implies A^n = PD^nP^{-1}$